Modeling Pavement Noise with Different Tire Configurations using Coupled FEM/BEM Analysis

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ABSTRACT

This paper aims to predict tire-pavement interaction noise using a coupled finite element modeling (FEM) and boundary element modeling (BEM) approach. A dynamic tire-pavement interaction model was first developed using the FEM to obtain tire vibration under harmonic excitations. The FEM simulation results provide the tire acceleration histories that serve as the boundary condition for predicting the exterior acoustic field of tire using the BEM. The developed FEM/BEM model was used to study the impact of dual tires and wide-base tires on pavement noise. Two radial truck tires (275/80R22.5 and 445/50R22.5) were modeled and the tire models were validated with tire deflections and contact stresses at the tire-pavement interface. The predicted contact stresses show different non-uniform distribution patterns between the dual-tire assembly and the wide-base 445 tire. Results indicate that the wide-base 445 tire was quieter than the dual tires and the differences in A-weighted sound pressure levels were observed up to 8.6 decibels (dB). The results illustrate the potential of using the developed model to capture the influence of tire types or tire dimensions on tire-pavement interaction noise.

INTRODUCTION

In recent years, the trucking industry has developed innovations of tire technology for purposes of improving the efficiency of vehicle operations and fuel consumption while minimizing tire wear. An increased interest in the use of wide-base single tires as an alternative to conventional dual-tire assemblies has occurred parallel to the recent interest as an innovative tire technology. Historically, dual-tire assemblies have been used to provide the largest footprint to adequately distribute the axle load onto the pavement surface. However, compared to conventional dual-tire assembly, wide-base tires has the potential to offer several benefits, including improved fuel efficiency, increased hauling capacity, reduced wheel cost, and superior ride and comfort (*1*).

The effects of wide-base tires on the road infrastructure is receiving considerable attention and eliciting widespread interest in pavement researchers, particularly since the introduction of new generation of wide-base tires (wide-base 445 and 455) in 2000s. The impact of the new generation of wide-base tires on pavement damage was firstly investigated in 2000. A comprehensive study was conducted to compare the pavement responses under wide-base tires and the dual-tire assembly on the heavily instrumented Virginia Smart Road. A number of studies were conducted to investigate the pavement damage mechanisms induced by different tire configurations using theoretical modeling and field instrumentations. These studies concluded that the new wide-base 445 or 455 tire could cause greater or less pavement damage potential than the dual-tire assembly, depending on the pavement structure and failure mechanism (*2-6*).

The aforementioned studies focus on the impact of wide-base tires on pavement damage. However, researches on the impact of wide-base tires on environment, especially the influence on tire-pavement noise is quite limited. Noise, defined as unwanted or excessive sound, has become one of the greatest sources of nuisance and environmental exposures in the United States (*7*). While noise emanates from different sources, traffic noise is perhaps the most pervasive and difficult source to avoid (*8*). In 1981, the U.S. Environmental Protection Agency (EPA) estimated that approximately 100 million people (about 50% of the population) in the United States had annual exposures to traffic noise that were high enough to threaten the health (*9*).

Traditionally, traffic noise is related with exhaust and engine noise of vehicles. With the development of automobile technology, the emission and propagation noise from such sources are greatly controlled. Instead, the tire-pavement interaction noise becomes dominant. As a major component of the vibrating source of the tire-pavement noise, the tire plays a significant role in noise control (*10*). The literature survey shows that most researchers consider the effect of tire dimensions on tire-road noise via the use of large-scale field or laboratory tests (*11*). These test methods consume considerable manpower, materials and financial resources. The testing results are susceptible to the interference from a variety of factors, such as environmental factors, traffic conditions, etc. Hence, it is desired to develop a numerical simulation method for evaluating the impact of new generation wide-base tire on tire-pavement noise, as an important means of developing wide-base tire.

OBJECTIVE AND SCOPE

This study explores the application of a modeling approach to predict tire-pavement noise level with different tire configurations. A coupled BEM/FEM analysis approach was

developed for prediction of tire-pavement interaction noise. Initially, a radial truck tire was first built using the FEM and the surface accelerations under harmonic excitations were obtained from the modal analysis. Then the solutions for the radiation sound fields caused by the tire vibration were solved using the BEM analysis. The tire model was validated with tire deflection and contact stresses at the tire-pavement interface. The FEM/BEM model was used to predict the sound pressure level generated by two truck tire configurations (275/80R22.5 and 445/50R22.5) on the porous asphalt concrete (PAC). Numerical predictions were compared with existing experimental observations from literature and showed consistent trends.

SIMULATION OF TIRE VIBRATION RESPONSES

To apply a tire model to tire-pavement interaction noise prediction, a model needs to be efficient yet accurately represents the tire structure behavior. Modern tires are structurally complicated, consisting of layers of belts, plies, and bead steel imbedded in rubber. The tire industry developed plentiful simplified models to predict tire performance, among which are the classical spring-damper model, the tire ring model and the membrane and shell model (*12*). Among numerous notable published contributions to tire modeling are publications by Pottinger and Joseph (*13-14*). Ahmad (*15*) developed a non-linear finite element model of the interaction of a tractor tire with soil surface for analyzing the contact pressure distribution for varying load conditions. Models commonly used for tire design purposes can accurately simulate tire performance, and the interaction of internal components. However, such models are extremely large and require long computing time. Besides, the tire material properties and related modeling details for tire models used by the industry are extremely difficult to obtain. Since the concern in this paper focuses on the deformation as it relates to the contact area and its ability to obtain modal characteristics, a simpler model can be employed for better computational efficiency.

Descriptions of Tire Finite Element Model

In this paper, two truck radial tires (275/80R22.5 and 445/50R22.5) were built under the same inflation pressure 724kPa, with load of 17.8kN and 35.6kN respectively, and a friction coefficient of 0.55. The nomenclature of tires usually includes three tire dimensions and types of tire in the form of AAA/BBXCC.C (*16*). The first number (AAA) is the tire width from wall to wall in mm or inch, the second number (BB) is the sidewall height given as a percentage of the tire width. The letter (X) indicates the type of tire (radial or bias ply). The third number (CC.C) is the tire rim diameter in inches. For example, a tire designation 445/50R22.5 is a radial tire (indicated with the “R”), with a wall-to wall width of 445mm, a wall height of 222.5mm, and a rim diameter of 22.5in (571.5mm). The two tire configurations (*17*), single wide-base and standard dual, are shown together in Figure 1.

Previous studies have pointed out that a tire model should consider the following main characteristics: 1) the composite material and the anisotropy resulted from the stiffness difference between rubber and reinforcement; 2) the relatively large deformation due to flexibility of tire carcass during tire pavement interaction; and 3) the nearly-incompressible and nonlinearity of rubber (*18*). The choice of material model depends on balancing the type of behavior desired in the model with the information available to determine tire model parameters. The tread and sidewalls are constructed from rubber,

and the belts and carcass are made of fiber-reinforced rubber composites. Both the rubber and fiber reinforcement are modeled as linear elastic material. Generally, the tire simulation consists of mounting the tire on the rim, followed by inflating it, lowering the tire onto a non-deformable surface, and applying various loading conditions.

FIGURE 1. Wide-base single (445/50R22.5) and on tire of dual-tire assemblies (275/80R22.5).

The simulation of FE tire model in this study includes three steps. In the first step, an internal inflation pressure is applied on the axisymmetric tire model. The second step of the simulation is generation of a full three-dimensional tire model by using the symmetric results transfer and symmetric model generation (*19*). In the last step the static deformed shape of the pressurized tire under a vertical load was modeled. The road surface is modelled as analytical rigid body. Two-dimensional tire mesh and three-dimensional tire mesh are shown in Figure 2.

The contact between the tire and pavement surface consists of two components: one normal to the surfaces and one tangential to the surfaces. The treatment of the normal contact condition is to enforce impenetrability in the normal direction using the penalty method or Lagrange multipliers method. However, for the tangential interaction between two contacting surfaces, the typical Coulomb friction law is used. This model assumes the resistance to movement is proportional to the normal stress at an interface. In this case, the interface may resist movement up to a certain level; then the two contacting surfaces at the interface start to slide relative to each another. If the relative motion occurs, the frictional stress remains constant and the stress magnitude is equal to the normal stress at the interface multiplied by the friction coefficient.

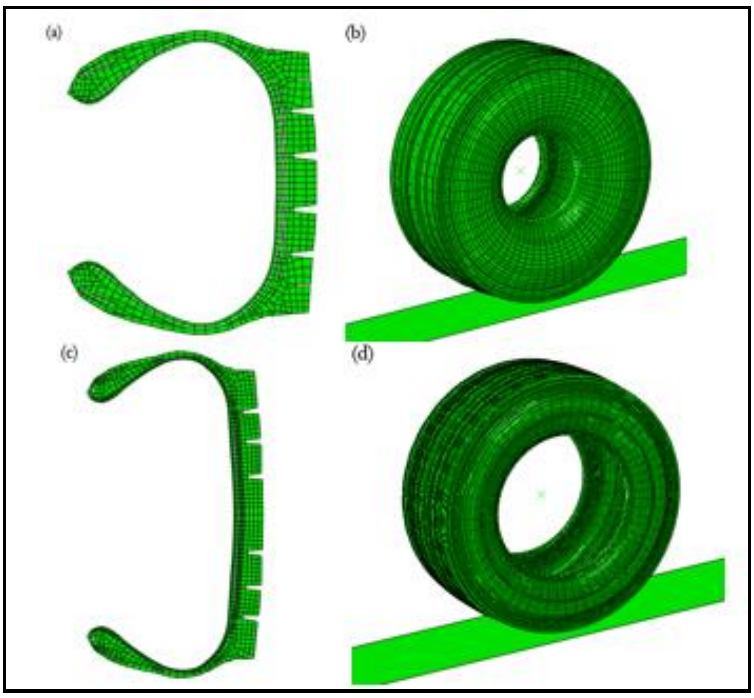
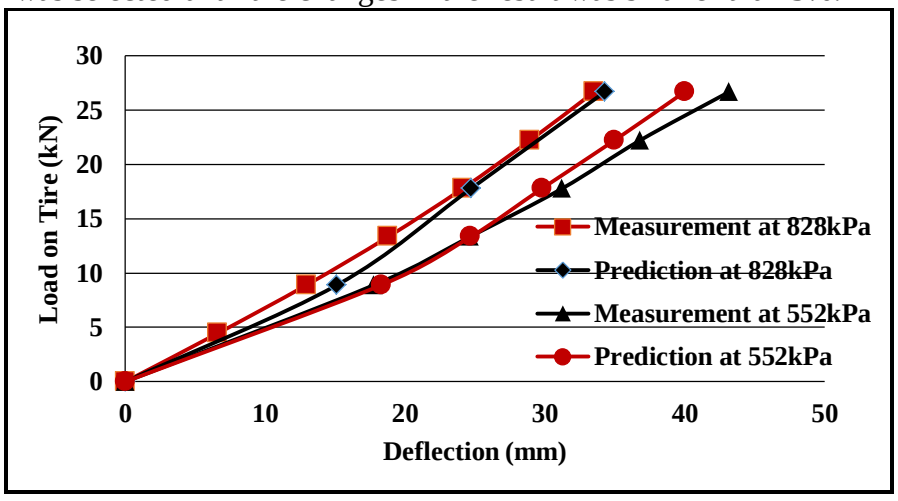


FIGURE 2. Tire finite element meshes (a) Two-dimensional 275 tire mesh; (b) Three-dimensional 275 tire mesh (c);Two-dimensional 445 tire mesh; (d) Three-dimensional 445 tire mesh.

Validation of Tire Finite Element Model

Tire deflection is an important measure of the tire stiffness in response to the vertical load. In this paper, tire load deflection curves and measured contact stresses curves from experimental measurements are used to calibrate the tire model (*20, 21*). Good agreements were achieved between the calculated tire deflections and measurements under different load and pressure conditions, as illustrated in Figure 3. A mesh sensitivity analysis with a series of gradually finer finite element meshes was conducted to refine the mesh in the contact area. By comparing the predicted contact stresses for each mesh, the final mesh was selected until the changes in the result was smaller than 5%.



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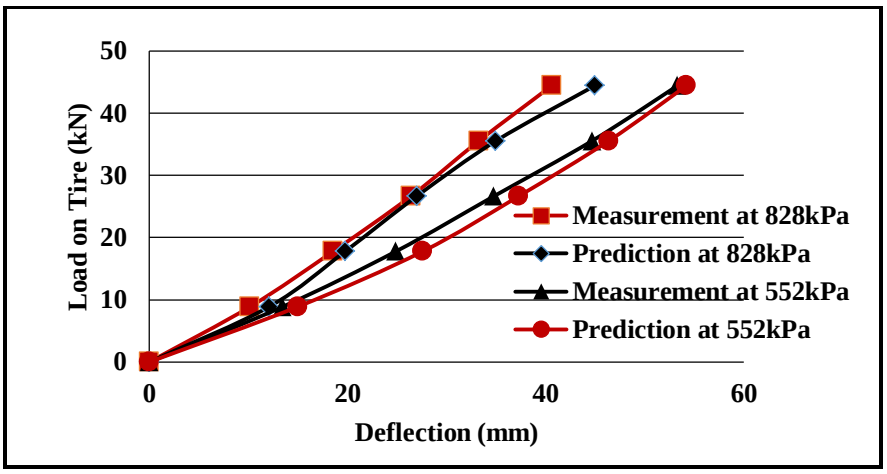
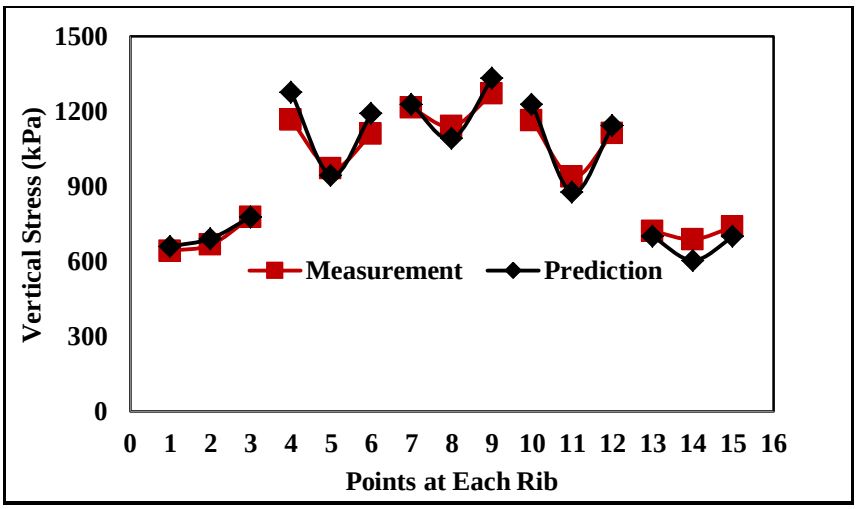


(b)

FIGURE 3. Comparisons between measured and predicted deflections for (a) dual 275 tire and (b) wide-base 445 tire.

The predicted vertical contact stresses at the tire-pavement contact zone were compared to the experimental measurements provided by the tire manufacturer. As shown in Figure 4, the tire model captured the non-uniform distribution of vertical contact stresses under each individual tire rib. As the tire is pressed against a flat surface, the tread rubber is compressed in the flattened contact patch and the sidewall of the tire is in tension. The bending stress in the sidewall causes the non-uniform distribution of vertical contact stresses in the contact patch, particularly at the edge of the contact patch. The maximum vertical stress under the center rib was found to be 1.4-1.6 times the inflation pressure owing to the non-uniform contact stress distribution.

The results indicate that the wide-base tire has relatively more uniform vertical stress distribution along the tire width compared to the dual-tire assembly, which is consistent with the measurements reported in the authors' previous work (*4*). It is noted that the non-uniform contact stresses have significant influence on pavement responses at the near-surface, as indicated in the previous work by the authors (*5, 6*). Based on the good agreement in the measured tire deflections and contact stresses, the proposed tire model is assumed to be reasonable for tire-pavement interaction analysis.



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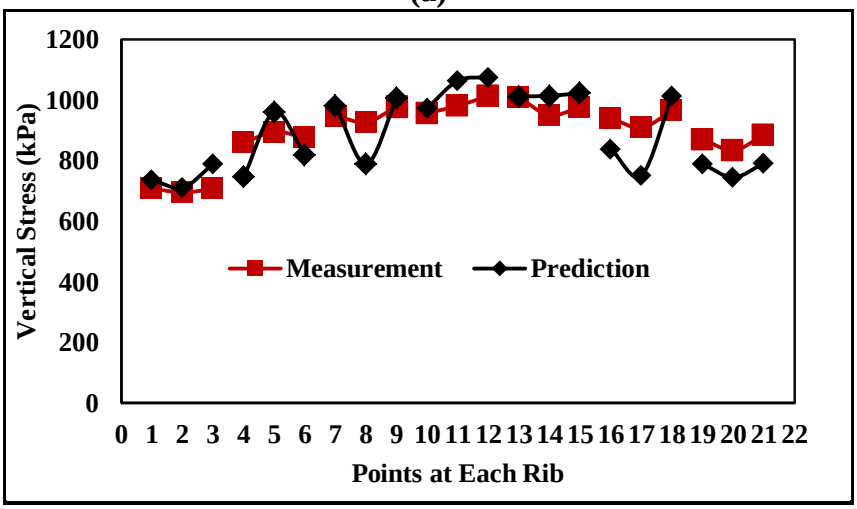


(b)

FIGURE 4. Comparisons between measured and predicted vertical contact stress for (a) dual tires and (b) wide-base tire.

Tire Modal Shapes and Vibration Responses

Modal analysis was conducted to identify the natural frequencies and mode shapes of the tire. The BEM requires a tire geometry and the surface acceleration to predict the acoustic pressure and intensity. The surface acceleration is obtained from the modal characteristics of a tire under harmonic excitation. The eigenvalue problem for natural modes of small vibration of a FEM model is shown in Equation 1.

$$\left(u^2[M]+u[C]+[K]\right)\{\phi\}=0 \tag{1}$$

where, $[M]$ is the mass matrix; $[C]$ is the damping matrix; $[K]$ is the stiffness matrix; u is the eigenvalue; and $\{\phi\}$ is the eigenvector, the mode of vibration. Typically, for symmetric eigenproblems, $[K]$ is assumed positive semidefinite. In this case u becomes an imaginary eigenvalue, $u = iw$ where w is the circular frequency and the eigenvalue problem can be simplified as Equation 2.

$$\left(-w^2[M]+[K]\right)\{\phi\}=0 \tag{2}$$

Two approaches can be used for solving the symmetrized eigenproblems: subspace iteration method and Lanczos method. Both methods use the Householder and Q-R algorithm for the reduced eigenproblem. The basic idea of the subspace iteration method is simultaneous inverse power iteration. In this method, a small set of base vectors is generated, which defines a "subspace". Then the "subspace" is transformed, by iteration, into the space containing the lowest few eigenvectors of the overall system (*22*). The Lanczos procedure consists of a set of Lanczos "runs", in each of which a set of steps is performed. The Lanczos eigensolver is a powerful tool for extraction of the extreme eigenvalues and the corresponding eigenvectors of a sparse symmetric generalized eigenproblem (*23*). This paper employs Lanczos method for tire modal analysis. There were a total of 20 structural vibration modes identified from the FEM modal analysis in the frequency range between 10-800 Hz. Figure 5 illustrates the first four radial modes of tire vibration, respectively, for one tire in dual-tire assembly and the wide-base 445 tire.

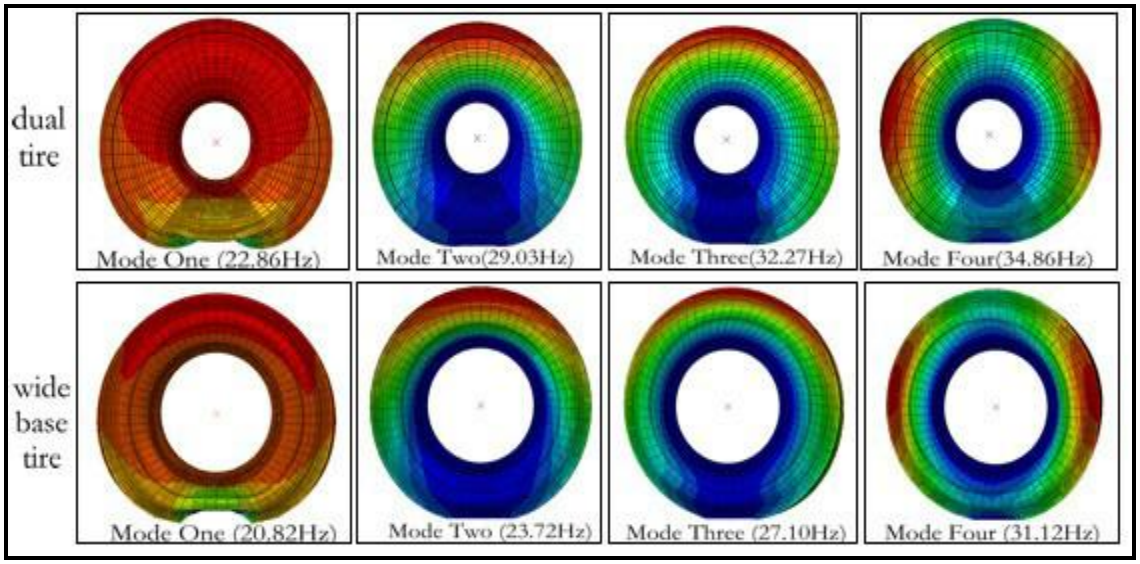


FIGURE 5. Finite element modal analysis results.

Steady state linear dynamic analysis predicts the linear response of a structure subjected to continuous harmonic excitation. The set of eigenmodes extracted in the previous eigenfrequency step were used to calculate the steady state solution as a function of the frequency of the applied excitation. The structural dynamics equation for steady-state response under a harmonic excitation is shown in Equation 3.

$$\left(\left[K\right]+iw\left[C\right]-w^2\left[M\right]\right)\left\{U\right\}=\left\{F\right\} \quad (3)$$

where, $\left[U\right]$ is the nodal displacement vector of the structure; and $\left[K\right]$ is the load vector of the external excitation.

The acceleration response obtained in the FE analysis was used as a boundary condition for a BEM analysis of the transmitted noise field. Generally, tire-pavement noise can be described as two mechanisms: the mechanical vibrations of the tire, and the aerodynamic phenomenon (*10*). In this study, only the vibrations noise is taken into consideration. Since the frequency spectrum characteristic of noise is not affected by the amplitude of the harmonic excitation, the acceleration responses of the tire was calculated with harmonic point loads of unit magnitude in the frequency range from 10 to 800Hz.

The point load is applied to the analytical rigid surface through its reference node. Figure 6 illustrates the acceleration frequency response of a node from the tire sidewall.

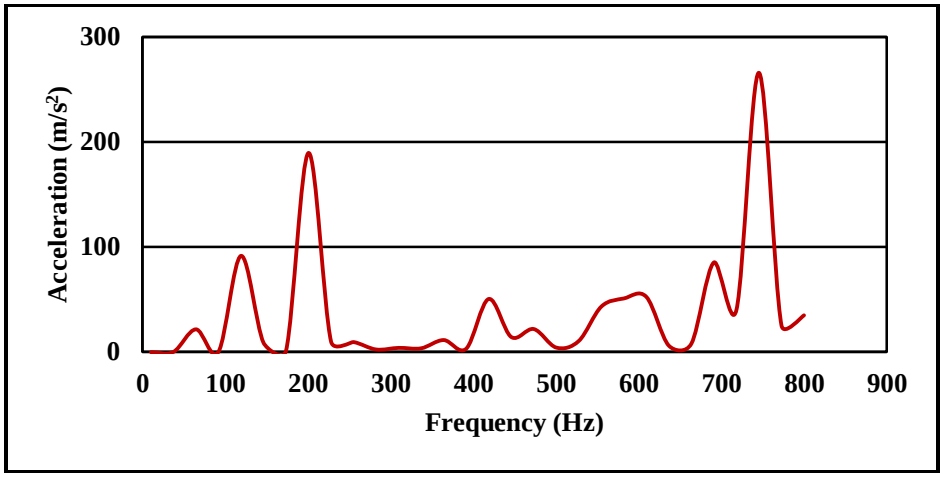


FIGURE 6. Example of tire acceleration spectrum at 37.24Hz for one tire in dual-tire assembly

TIRE-PAVEMENT NOISE PREDICTION

Basic Theory of BEM

The mathematical formulation and practical application of the BEM have been discussed in details in previous publications, such as Brebbia and Walker (*24*), Telles and Wrobel (*25*). The application of the BEM to wave problems can be found in the work of Shaw and Copley (*26, 27*).

As a numerical method used to solve a wide variety of boundary value problems, there are two approaches in the boundary element formulation: the direct boundary element method (DBEM) and the indirect boundary element method (IBEM). In the DBEM the physical variables, such as acoustical pressure and acoustical velocity are solved directly from the discretized surface integral equations. By comparison, in the IBEM the problem is formed in accordance with a source density function, and the distribution of fictitious sources is solved on the boundary. In this study, only the DBEM was used.

The governing partial differential equation for linear acoustics in the frequency domain is the Helmholtz equation, Equation 4.

$$\left(\nabla^2 + k^2\right)p = 0 \tag{4}$$

where, ∇^2 is the Laplace operator and p is the acoustic pressure. $k = w/c$ is the wave number. w is the angular frequency and c is the speed of sound. The boundary condition for the vibro-acoustic problems is given in Equation 5.

$$\partial p/\partial n = -i\rho w v_n \tag{5}$$

where, n is the normal vector pointing outside from the acoustic volume; $i = \sqrt{-1}$ is the imaginary unit; ρ is the acoustic fluid density and v_n is the normal velocity. From the theory of Green's functions, Equation 6 can be recast into the Helmholtz integral for an exterior boudnary value problem (*28*):

$$p(P) = \int_{\Gamma} \left(G \frac{\partial p}{\partial n} - p \frac{\partial G}{\partial n} \right) d\Gamma \tag{6}$$

where, $G = e^{-ikr} / 4\pi r$ is the singular fundamental solution, and r is the distance between the field point.

Boundary Element Modeling of Tire Pavement System

The acceleration response obtained in the FE analysis as described in the previous section is used as a boundary condition for a BEM analysis of the transmitted noise field. To solve the acoustic problem, the DBEM formulation was used in this study. For a given acceleration field on the tire BEM surface, an acoustic BEM direct frequency response analysis calculates pressure and normal acceleration values at all the boundary nodes and field points. The outer surface elements of the tire were selected to constitute the boundary element. In order to avoid sound leakage, additional elements were added to the rim. In the dual-tire model, a symmetric total reflection panel is built to simulate the other tire. The distance between the symmetric panel and tire is 310 mm (dual spacing). The boundary of the BE model is exactly the same as the FE model of the tire, in this way the acceleration response of the tire could be transferred directly to the BE model.

Tire-Pavement Noise under Different Tire Configurations

In this study, field points to calculate sound pressure are chosen to be identical to the measurement location in the CPX method. Instead of measuring sound intensity level from a phase-matched pair of microphones (as the OBSI method does), CPX measures levels via sound pressure from a single microphone. According to the ISO/DIS 11819-2, the microphone positions in the CPX method are illustrated in Figure 7. The five microphones in CPX method are modeled with corresponding five field points in BEM. Table 1 presents the exact positions of the field points in BEM. The coordinates of the origin is (0, 0, 0) located at the tire center.

TABLE 1 Field point coordinates for dual-tires and wide-base tire

Field Point	X Coordinate (mm)	Y Coordinate (mm)	Z Coordinate (mm)
Dual Tires			
1	0	337	265
2	200	337	265
3	-200	337	265
4	650	0	165
5	-650	0	165
Wide-base Tire			
1*	0	422	435
2*	200	422	435
3*	-200	422	435
4*	650	0	335
5*	-650	0	335

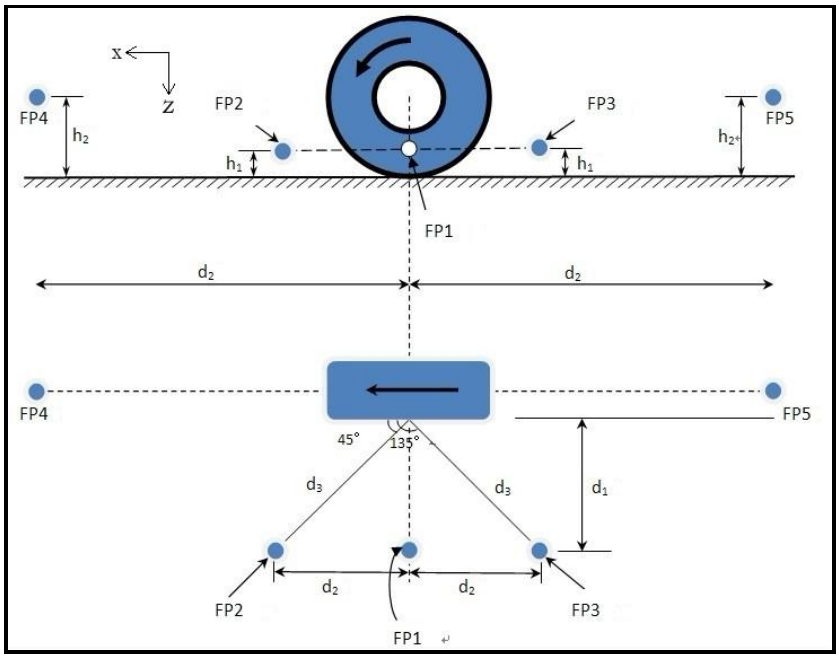
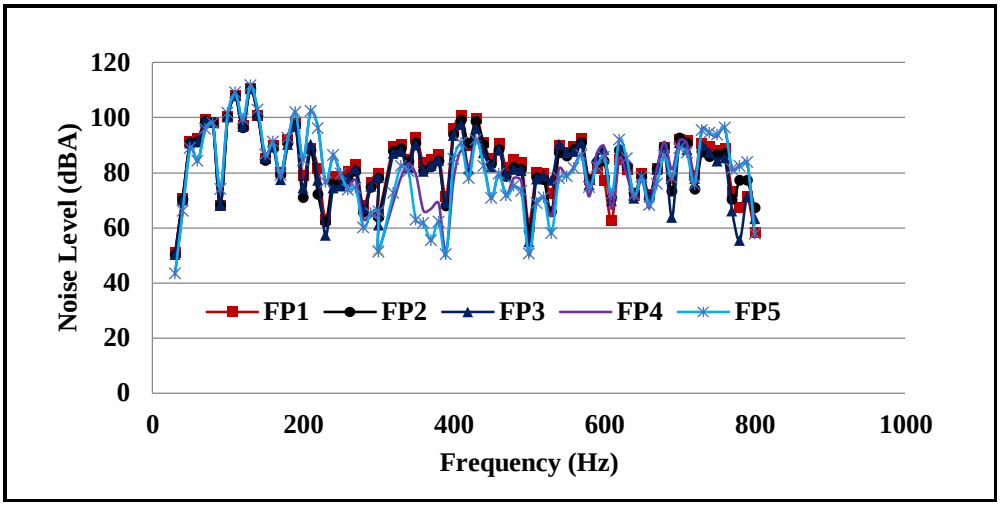


FIGURE 7. Microphone positions in the CPX method.

The road surface in the BEM was modeled as porous asphalt concrete (PAC) surfaces. PAC consists primarily of gap-graded aggregates held together by a polymer modified binder to form a matrix with inter-connecting voids. Unlike conventional asphalt pavements, PAC typically possesses 15-25 % void volume (*11*). In effect this increase in porosity means that the PAC acts as porous sound-absorbent materials and greatly absorbs the sound, converting it into a small amount of friction energy, thereby reducing noise. To theoretically model the sound propagation through a porous absorber, it is necessary to have frequency dependent sound velocity, and frequency dependent mass density which characterize the acoustic properties of the absorbent acoustic medium. In our previous work, a theoretical model has been developed to analyze the effects of pore structure parameters (pore radius, pore length and porosity) on the acoustic absorption coefficient of PAC. The porous material parameters with pore radius 6mm, pore length 4cm and porosity 25% used in this study are obtained from our previous work (*29*).

Figures 8 (a) and (b) illustrate the frequency responses of the sound pressure level at the field points, respectively, for the dual-tire assemble and the wide-base tire. In the figures, there are several dominant peaks of sound pressure responses at different frequencies. The simulated tire-pavement noise peak is higher than the present experimental data and the trend of the noise level curves have more fluctuations compared to the measured data from existing literatures (*30-31*). That may be partly attributed to the effects of the simple harmonic excitation and factors related to tire parameters like tire material, tire dimension. Other factors like temperatures and the rolling condition of tire could also affect the noise level.

It was observed from the figures that the wide-base tire induces smaller sound pressure level at the low frequency range (10-200Hz). While within the frequency range higher than 200Hz, the wide-base tire causes the same noise level compared with the conventional dual tires. This probably because the acceleration response difference between these two tires as discussed in the above section.



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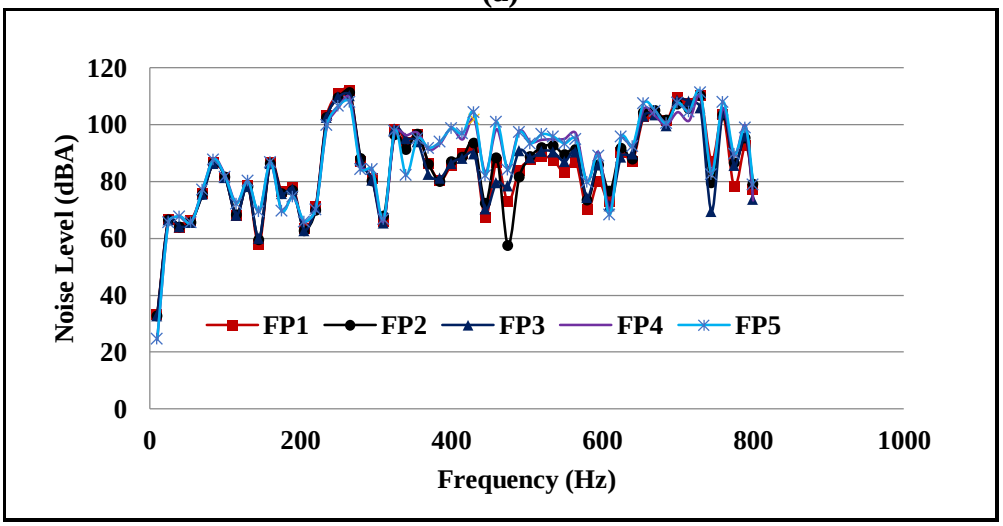

(b)

FIGURE 8. Sound pressure at selected field points for (a) dual-tire assembly and (b) wide-base tire.

The analysis in Figure 8 simulates sound pressure level at different frequency bands L_{pi}. The overall sound pressure level L_p is calculated as

$$L_p = 10 * \log_{10} \sum_{i=1}^{N} 10^{L_{pi}/10} \tag{7}$$

TABLE 2 Comparison of overall noise level at field points

Tire Type	FP1 (dBA)	FP2 (dBA)	FP3 (dBA)	FP4 (dBA)	FP5 (dBA)
Dual-tire assemblies	125.5	125.1	125.0	126.4	126.4
Wide-base tire	118.2	117.5	117.0	116.7	117.8

The overall noise levels for these two kinds of tires at field points are presented in Table 2. Concerning the impact of wide-base tire and dual tires on overall tire pavement interaction noise level, it has been shown here that the wide-base tires is quieter than the

dual tires. Differences in A-weighted sound pressure levels up to 8.6 dB(A) are observed. One possible reason why dual-tires seem to emit higher noise levels may be that the number of tires is higher. As literature (*11*) showing 16 wheels of a typical tractor-semitrailer combination give 6 dB higher noise level than a reference four-wheel vehicle. Further investigation is needed to validate the finding using field noise measurements for different truck tires.

CONCLUSIONS

In this study, a coupled FEM and BEM model was developed to predict tire-pavement interaction noise. The results show that the sound field caused by tire vibration and absorption properties of pavement surface can be predicted with appropriate selection of the excitation spectrum and acoustic parameters of pavement surface using the developed approach. The FEM/BEM model was used to predict the noise generated by the two truck radial tires. The results show that in the low frequency range lower than 200Hz, the dual-tire assembly is noisier than the wide-base tire. However, within the frequency range higher than 200Hz, the wide-base tire causes the similar noise level compared to the conventional dual tires. Concerning the impact of tire configuration on the overall tire-pavement interaction noise, it was found that the new generations of wide-base tire is quieter than the dual tires. Differences in A-weighted sound pressure levels were observed up to 8.6 dB(A). The results illustrate the potential of using the developed model to capture the influence of tire types or tire dimensions on tire-pavement interaction noise.

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