

Ground Vibration Prediction Uncertainties

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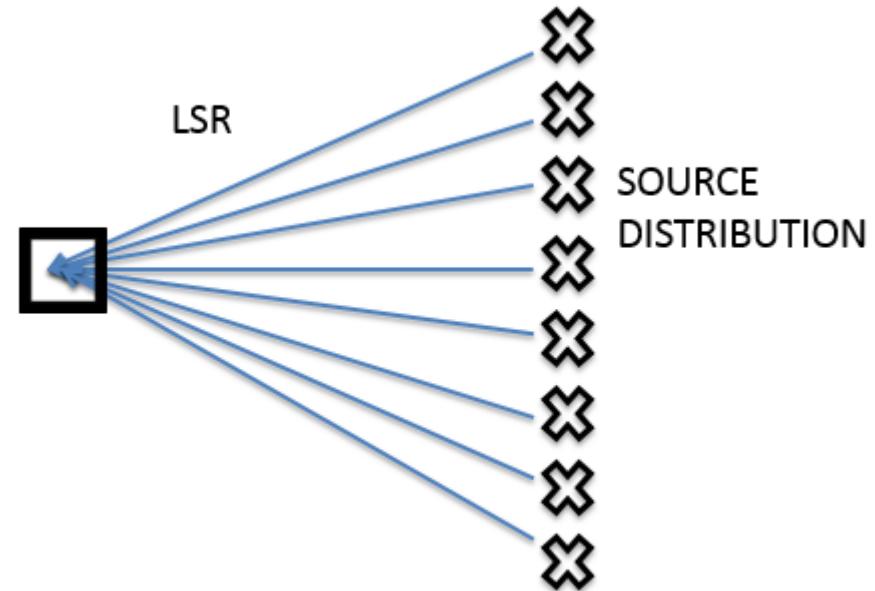
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WASHINGTON, DC



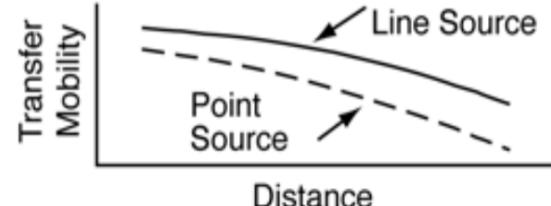
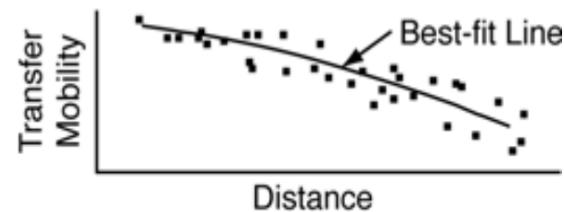
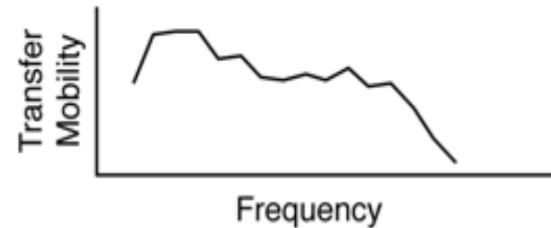
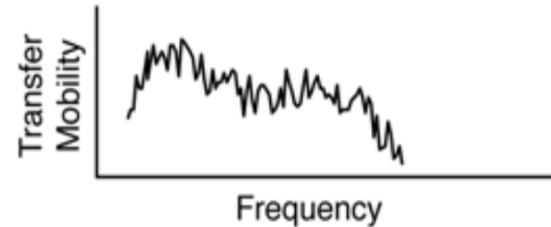
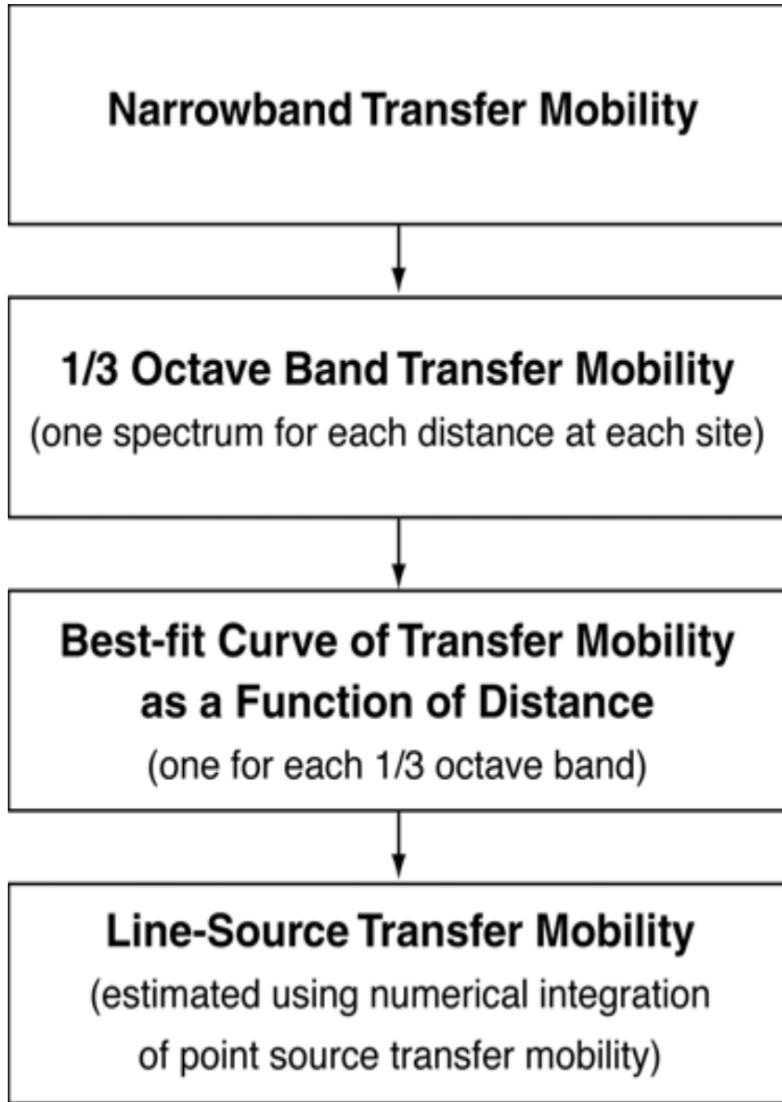
WILSON IHRIG
ACOUSTICS, NOISE & VIBRATION

FTA PREDICTION MODEL

- $L_v = FDL + LSR$
- L_v =Velocity Level
- dB
- FDL = Force Density Level - dB
- LSR = Line Source Response - dB



PROCEDURE



MEASUREMENT OF TM

- Relation between force F and velocity V

$$V(\mathbf{r}_2, f) = Y(\mathbf{r}_2 | \mathbf{r}_1, f) F(\mathbf{r}_1, f)$$

$$Y(\mathbf{r}_2 | \mathbf{r}_1, f) = \frac{\langle G_{FV}(f) \rangle}{\langle G_{FF}(f) \rangle} = \text{TRANSFER MOBILITY}$$

$\langle G_{FV}(f) \rangle = \text{CROSS SPECTRUM BETWEEN}$

$\langle G_{FF}(f) \rangle = \text{AUTO SPECTRUM FORCE}$

$\langle \rangle = \text{EXPECTATION VALUE}$

COHERENCE

- The coherence is the portion of vibration response energy that is related to the force

$$\gamma^2 = \frac{\langle G_{FV} \rangle \langle G_{FV} \rangle^*}{\langle G_{FF} \rangle \langle G_{VV} \rangle} = \frac{|G_{FV}|^2}{\langle G_{FF} \rangle \langle G_{VV} \rangle}$$

- The unrelated energy is noise energy that is a source of random uncertainty
- The coherence provides a measure of the uncertainty

STANDARD DEVIATION

$$s.d.[Y] = \sqrt{\text{var}(\text{Re } Y) + \text{var}(\text{Im } Y)}$$

$$\text{var}(\text{Re } Y) = \frac{1}{N_d} \sum_{d=1}^{N_d} (\text{Re } Y_i - \text{Re } \bar{Y})^2$$

$$\text{var}(\text{Im } Y) = \frac{1}{N_d} \sum_{d=1}^{N_d} (\text{Im } Y_i - \text{Im } \bar{Y})^2$$

N_d = Number of conversions (or samples)

STANDARD ERROR OF THE MEAN

$$s.e.\left[\bar{Y}\right] = \frac{s.d.\left[\bar{Y}\right]}{\sqrt{N_d}} = \frac{\sqrt{\text{var}(\text{Re } Y) + \text{var}(\text{Im } Y)}}{\sqrt{N_d}}$$

N_d = Number of conversions (or samples)

NORMALIZED ERROR OF MEAN COMPLEX MOBILITY

$$\varepsilon[Y] = \frac{\text{s.e.}[Y]}{|Y|}$$

$$\varepsilon[Y] = \sqrt{\frac{1 - \gamma^2}{(N_d - 1)\gamma^2}}$$

γ^2 = COHERENCE

N_d = NUMBER OF SAMPLES (CONVERSIONS)

NORMALIZED ERROR OF TRANSFER MOBILITY MAGNITUDE

- Bendat, JSV v59(3) 1978

$$\varepsilon_r[|Y|] = \sqrt{\frac{1-\gamma^2}{2N_d\gamma^2}}$$

γ^2 = COHERENCE

N_D = NUMBER OF SAMPLES (CONVERSIONS)

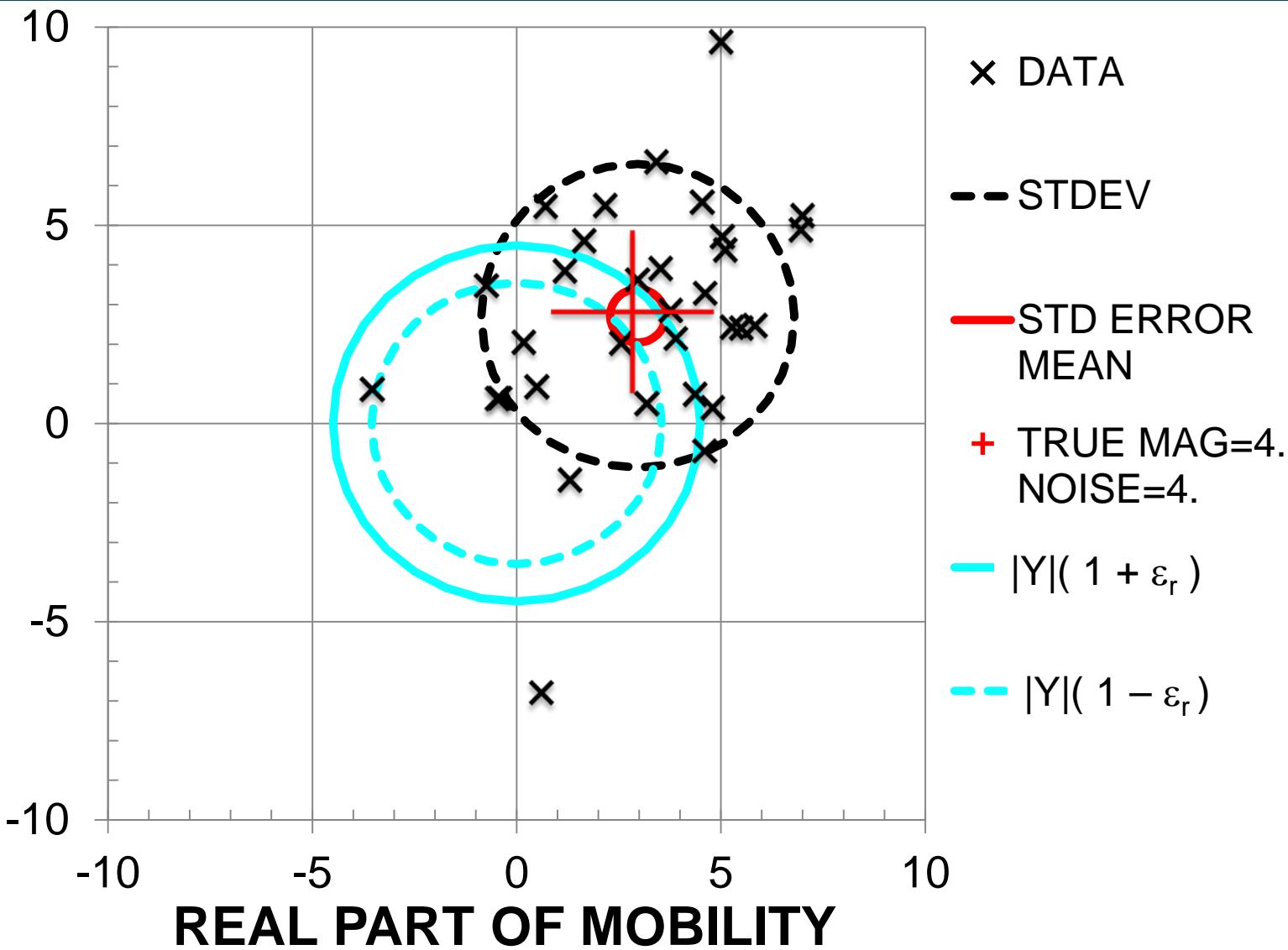
- Note: $\varepsilon[Y] > \varepsilon_r[|Y|]$

SIMULATION

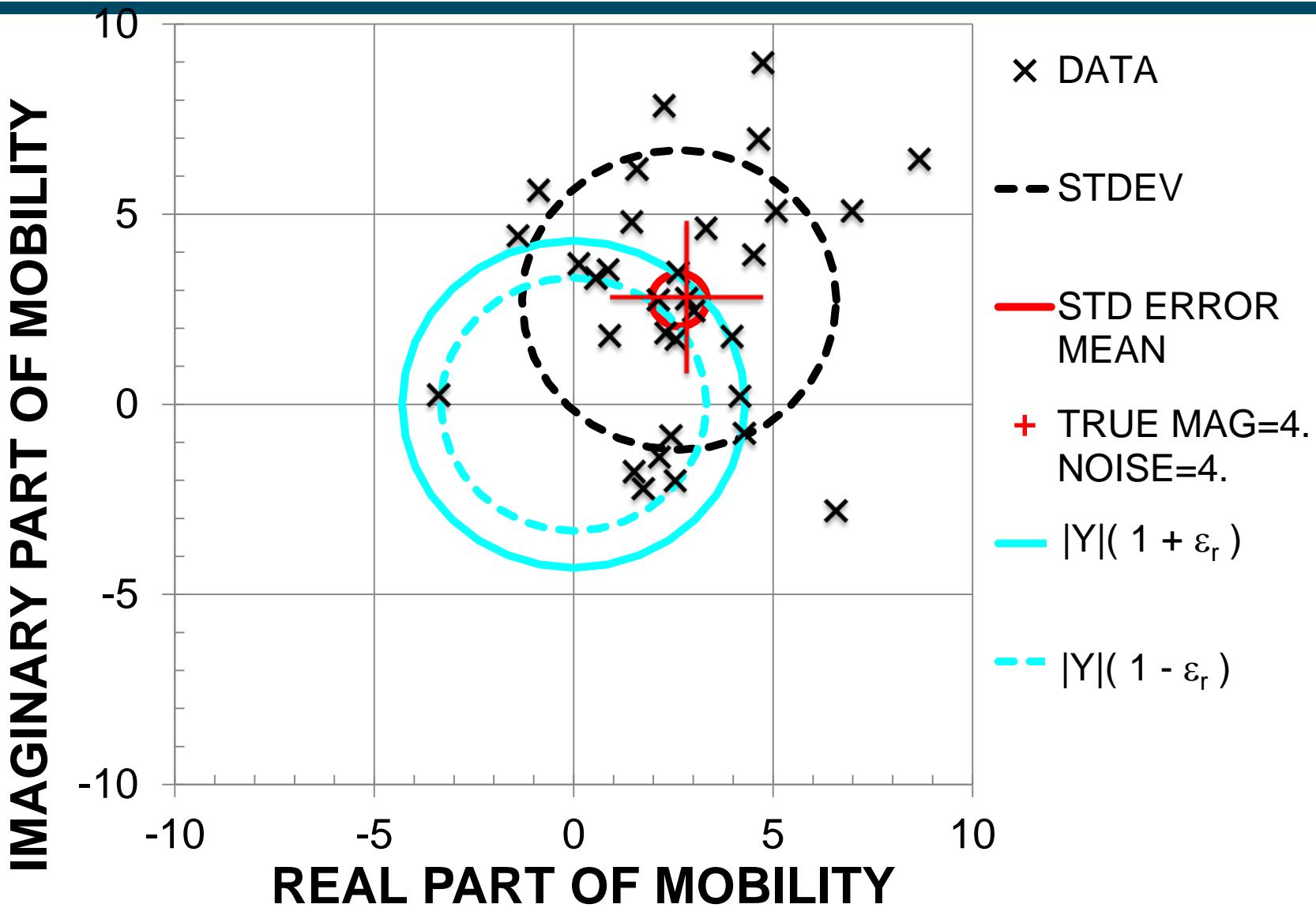
- Magnitude $|Y| = 4$
- RMS Noise = 4
- Unit force magnitude
 - $|GAA| = 1$
- 32 Conversions (eg Hammer Hits)

STDEV REAL=2.49 STDEV IMAG=2.91
COHERENCE = .532 TM EST ERR= 12%

IMAGINARY PART OF MOBILITY

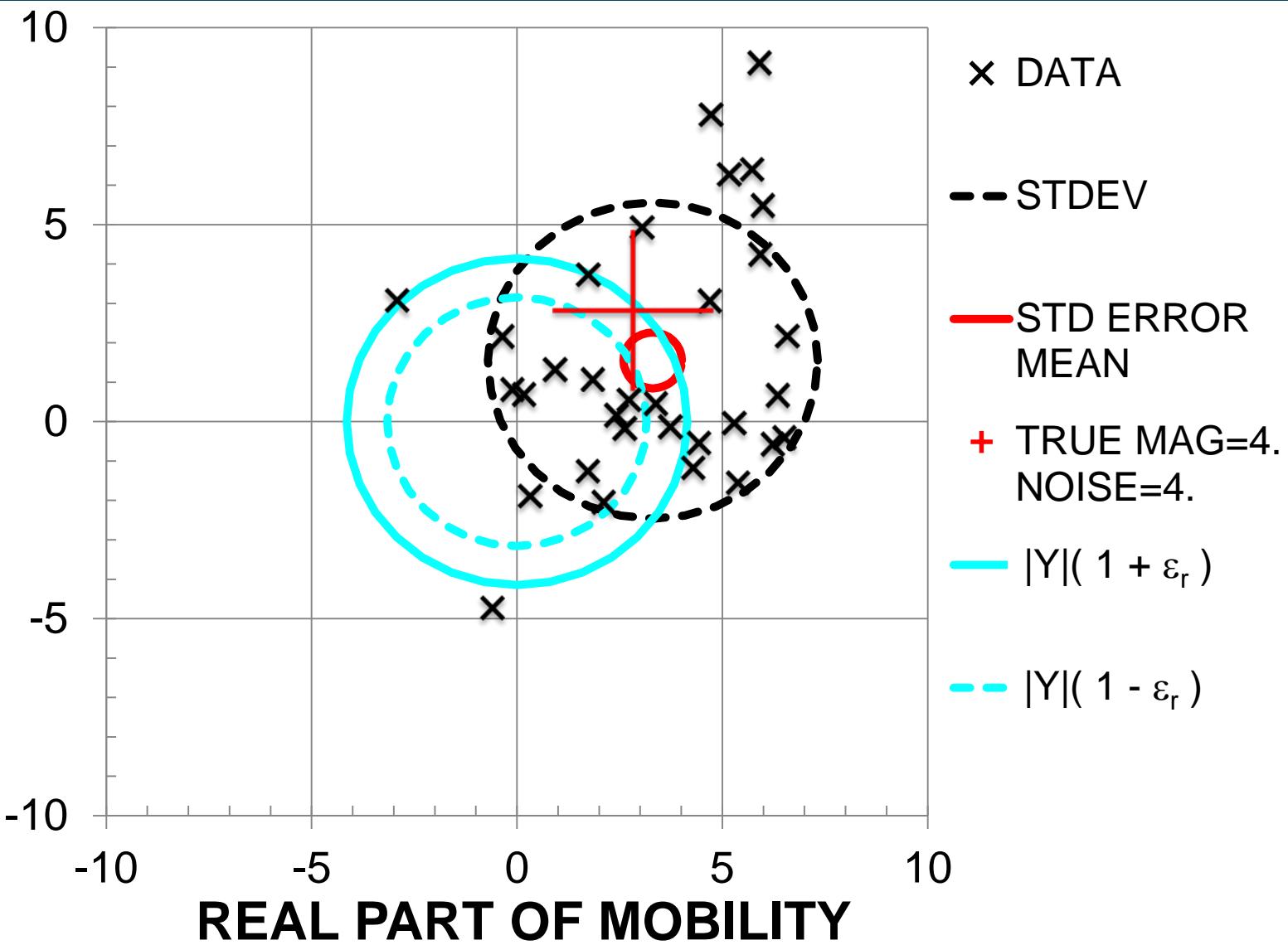


**STDEV REAL=2.43 STDEV IMAG=3.11
COHERENCE = .491 TM EST ERR= 13%**



STDEV REAL=2.52 STDEV IMAG=3.12
COHERENCE = .461 TM EST ERR= 14%

IMAGINARY PART OF MOBILITY



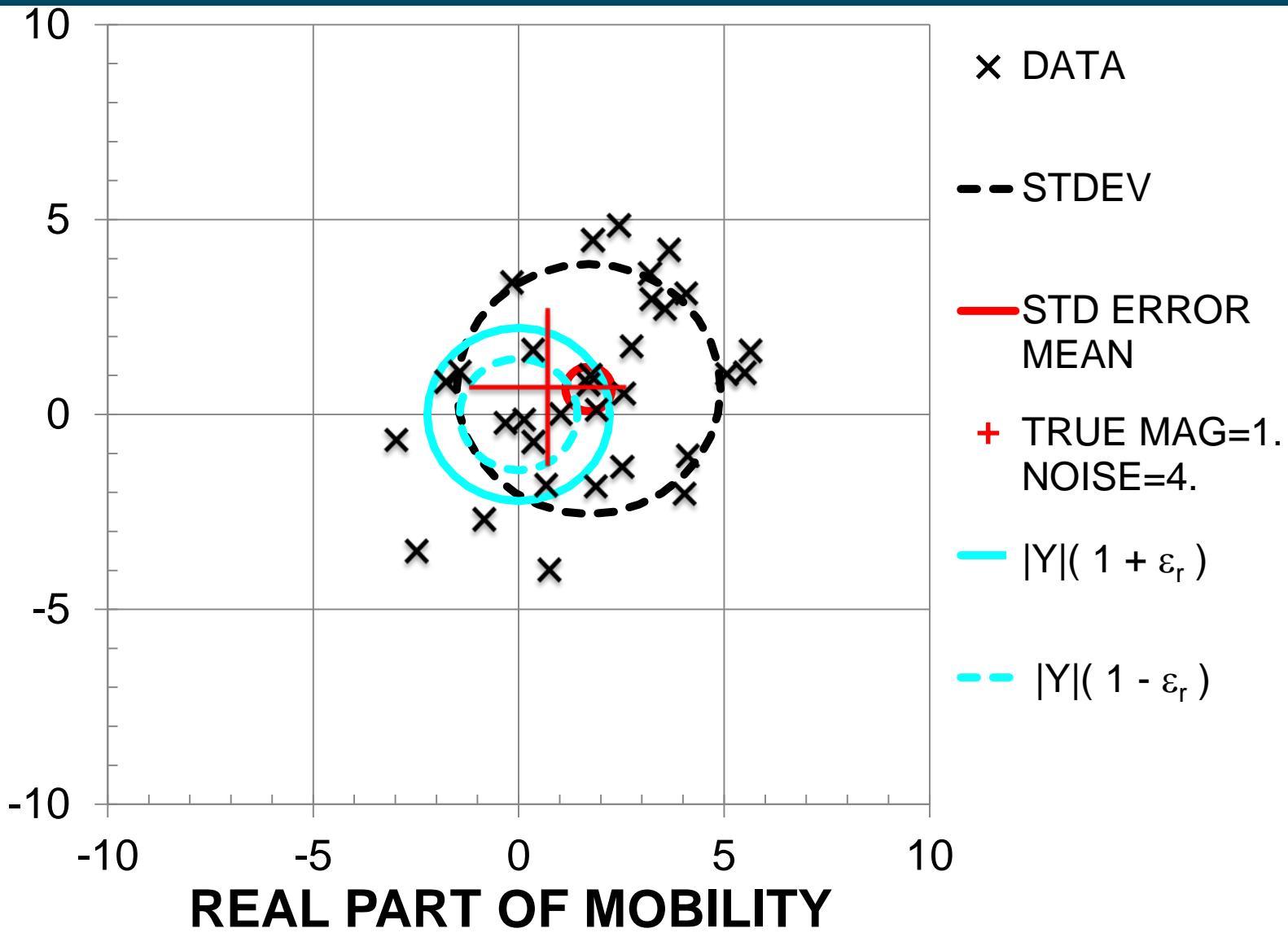
REAL PART OF MOBILITY

SIMULATION

- Reduce Magnitude $|Y|$ to = 1
- Keep Noise = 4
- Unit force magnitude
 - $|GAA| = 1$ for all frequencies
- 32 Conversions

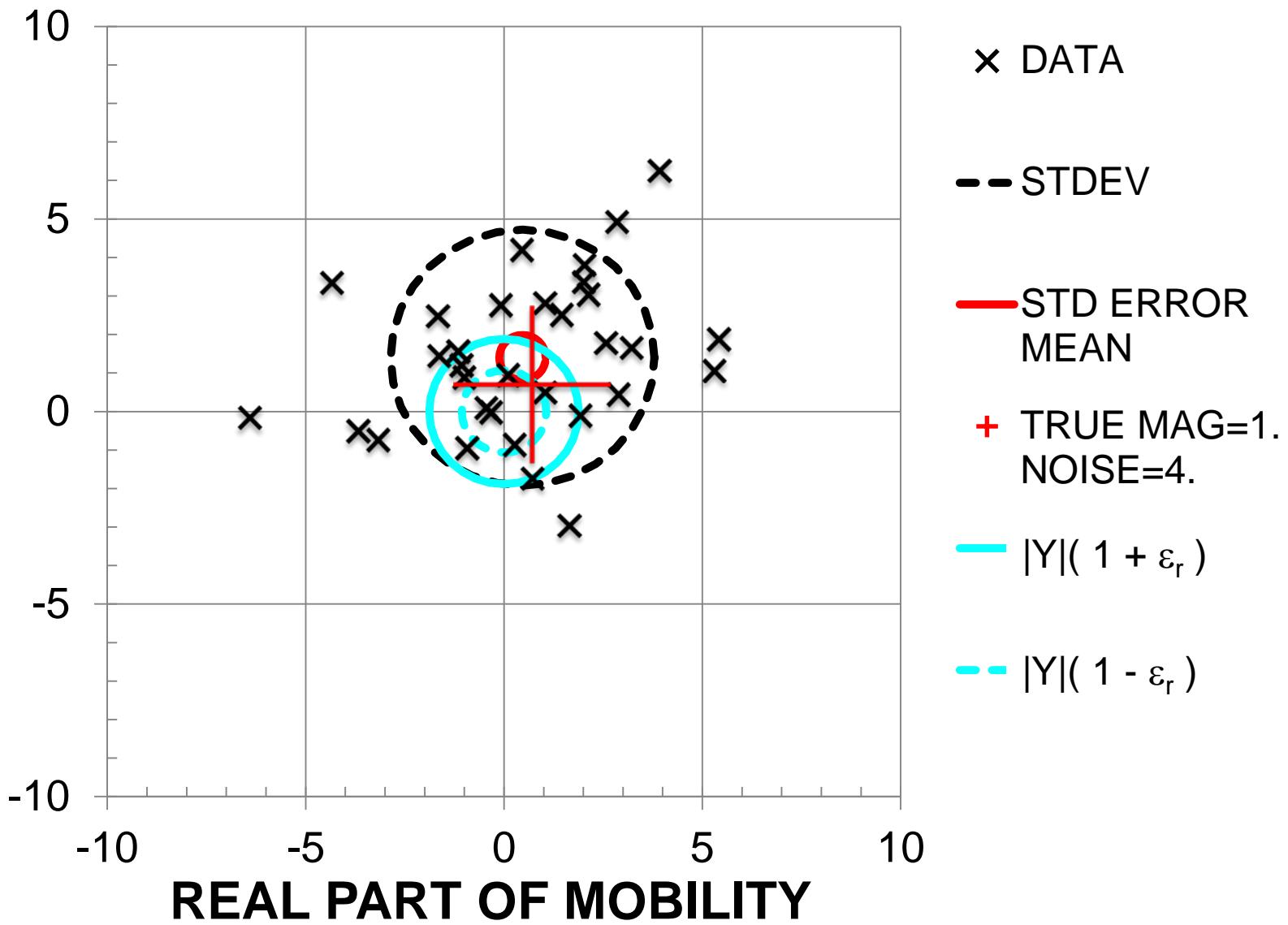
**STDEV REAL=2.25 STDEV IMAG=2.29
COHERENCE = .25 TM EST ERR= 22%**

IMAGINARY PART OF MOBILITY



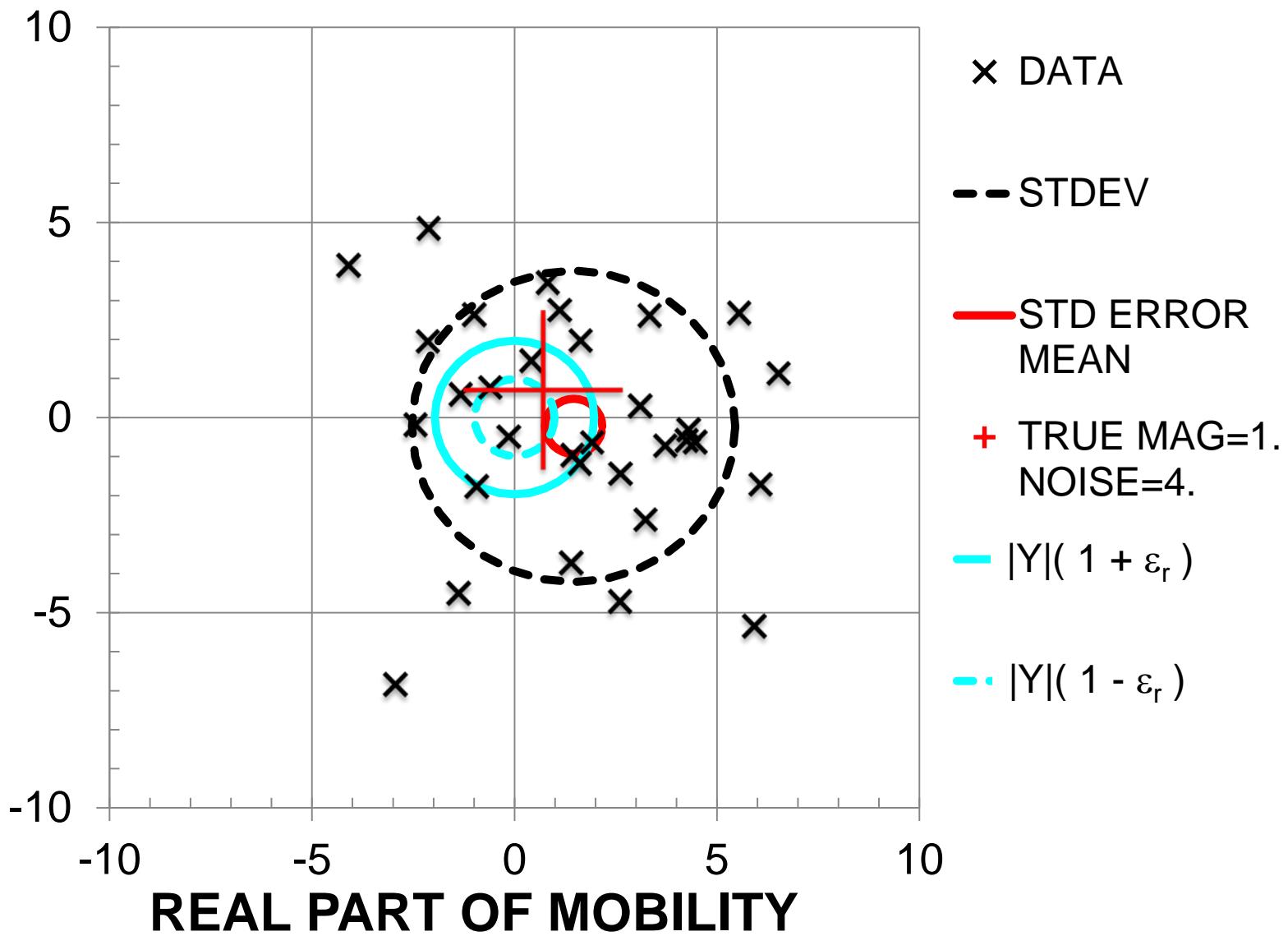
STDEV REAL=2.65 STDEV IMAG=2.01
COHERENCE = .169 TM EST ERR= 28%

IMAGINARY PART OF MOBILITY



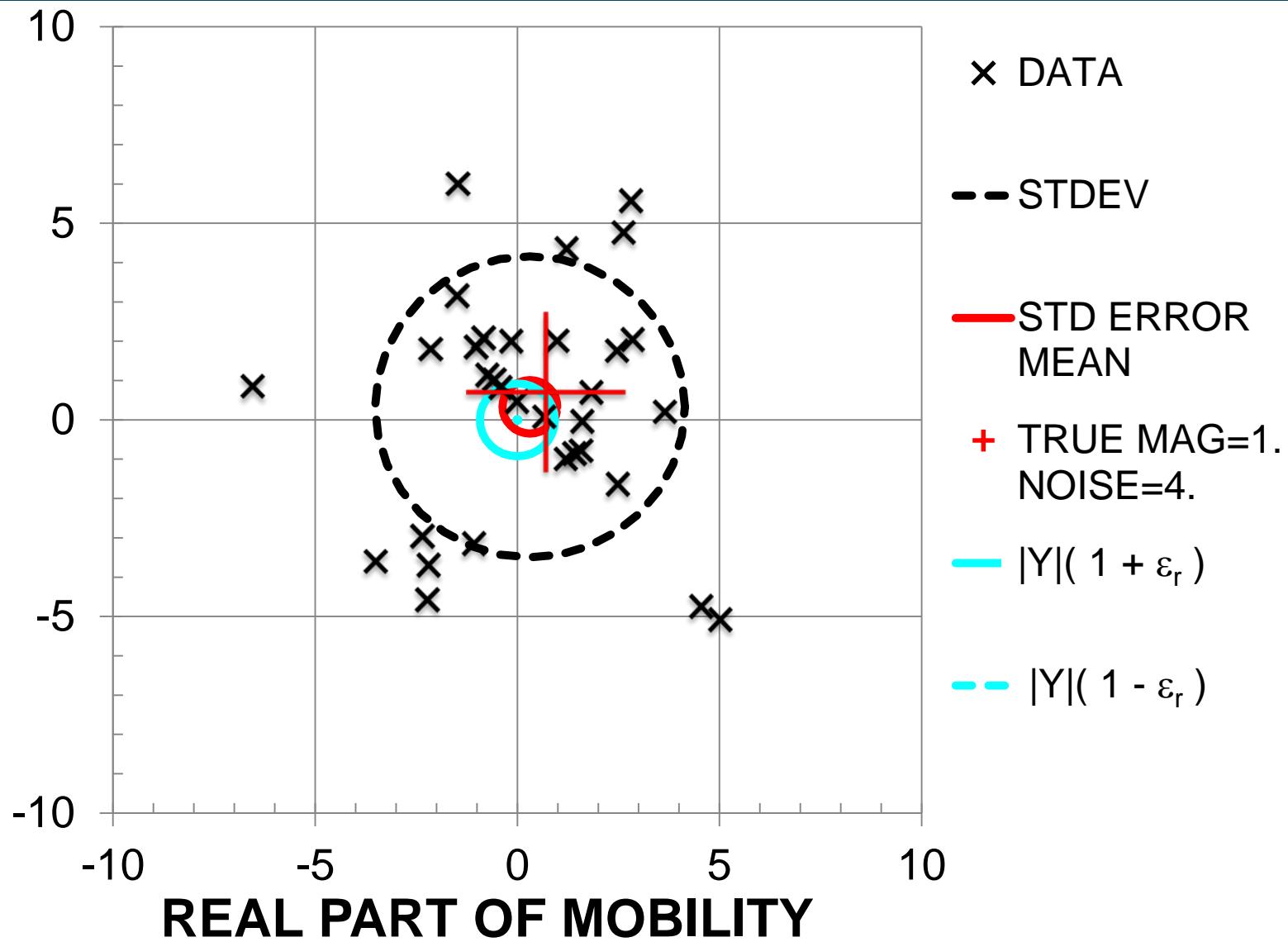
**STDEV REAL=2.86 STDEV IMAG=2.79
COHERENCE = .124 TM EST ERR= 33%**

IMAGINARY PART OF MOBILITY



**STDEV REAL=2.47 STDEV IMAG=2.93
COHERENCE = .014 TM EST ERR= 103%**

IMAGINARY PART OF MOBILITY



CONVERSION TO 1/3 OCTAVE MOBILITY LEVEL

- Average the square of the magnitude of Y over the 1/3 octave

$$TM_{1/3}(f) = \sqrt{\frac{\int |H(f)|^2 |Y(f)|^2 df}{\int |H(f)|^2 df}} = \text{1/3 octave mobility}$$

$H(f)$ = FILTER RESPONSE

f = FREQUENCY

- Energy averaging!

ESTIMATED NORMALIZED ERROR OF TM_{1/3}

$$\Sigma = \frac{\langle \varepsilon_r \rangle}{\sqrt{M}} = \text{Estimated Error of TM}_{1/3}$$

$\langle \varepsilon_r \rangle$ = Mean square estimated error of $|Y|$ for each bin

M = Number of frequency bins in the 1/3 octave band

$$\Sigma = \frac{1}{\sqrt{M}} \sqrt{\frac{1 - \gamma^2}{2N_d \gamma^2}} \quad \text{if all standard errors are the same}$$

REGRESSION vs DISTANCE

- Develop regression curve for integration of the Transfer Mobility over the train
- Most common
 - Level (dB) vs log distance
 - Level (dB) vs polynomial of log distance
 - Level (dB) vs log distance and distance

$$PSR(dB) = 20 \log_{10} [TM_{1/3}]$$

REGRESSION METHODS

- Linear Least Squares
- Chi-square minimization

$$\chi^2 = \sum_L \left(\frac{1}{\sigma_i^2} \left\{ PSR_i - \sum_{j=1}^L a_j X_j(x_i) \right\}^2 \right)$$

$\sigma_i(dB) = 20 \text{Log}_{10} [1 + \Sigma_i]$ = decibel uncertainty at each x_i
 x_i distances

PSR_i = transfer mobility level (dB) at x_i

$\sum_{j=1}^L a_j X_j(x)$ = regression function of x

a_j = regression coefficients

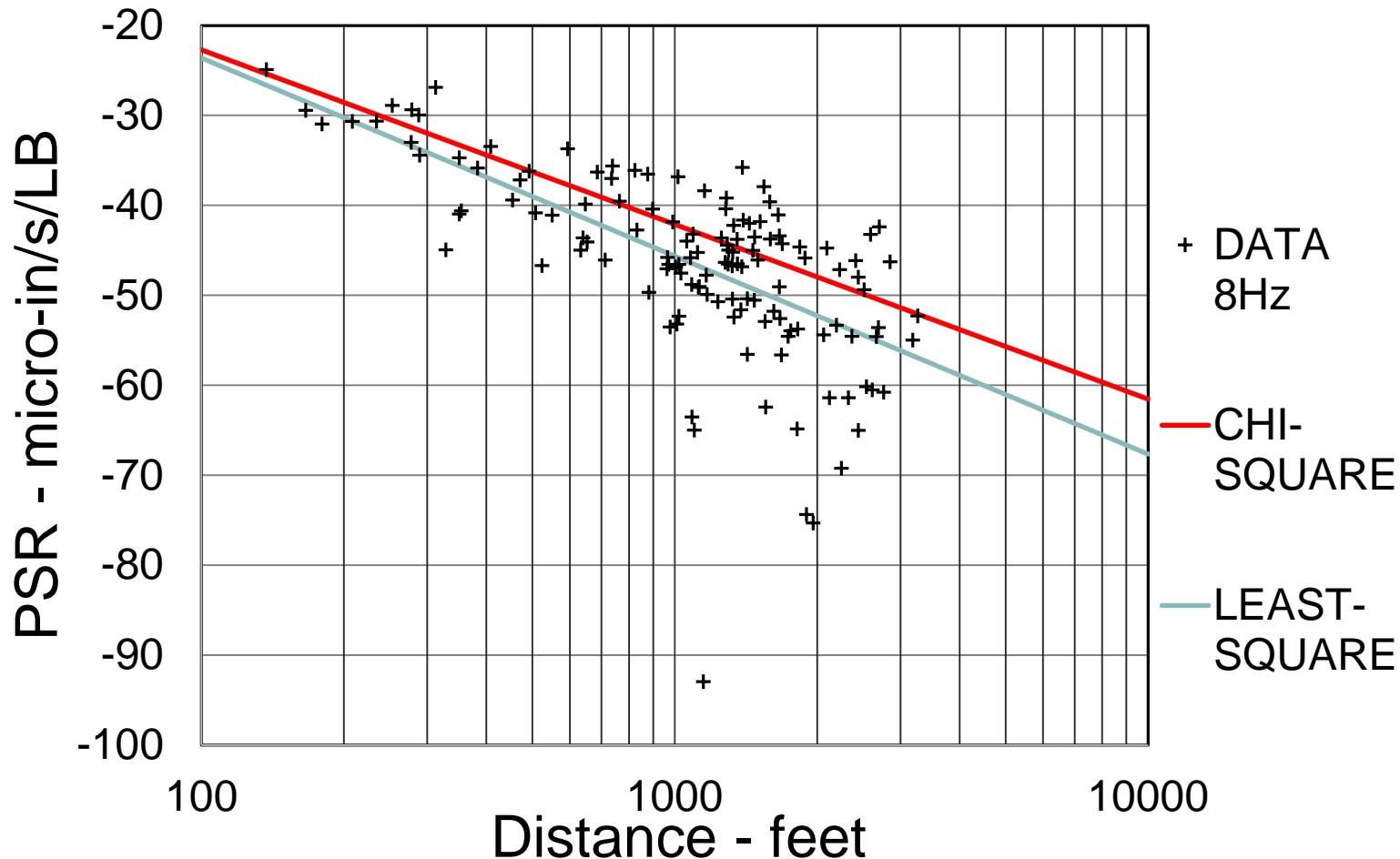
LEVEL UNCERTAINTY IN DB

- Practical relation for 1/3 Octave decibel uncertainty for chi-square regression

$$\sigma(dB) = 20 \text{Log}_{10} \left[1 + \Sigma_{1/3} \right]$$

EXAMPLE COMPARISON

- Linear regression vs chi-square regression



CONCLUSION

- Transfer mobility analysis with FFT provides sufficient data for estimation of error
 - Coherence function
 - Number of samples
 - Integration over 1/3 octave
 - Regression / Summation
- Easily incorporated into analysis software
- Risk assessment

QUESTIONS

