Ground Vibration Prediction Uncertainties

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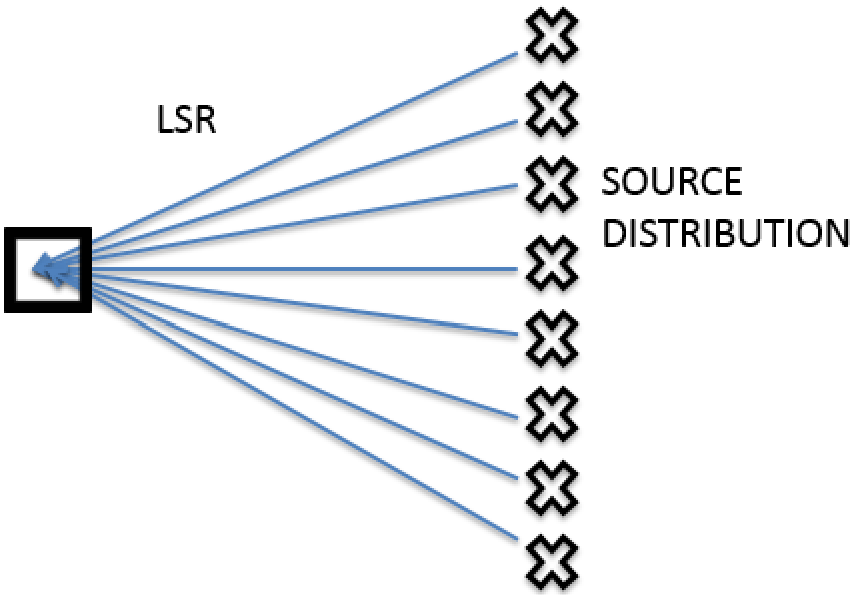
ACOUSTICS, NOISE & VIBRATION

FTA PREDICTION MODEL

- Lv = FDL + LSR
- Lv=Velocity Level - dB
- FDL = Force Density Level - dB
- LSR = Line Source Response - dB



PROCEDURE

Narrowband Transfer Mobility

↓

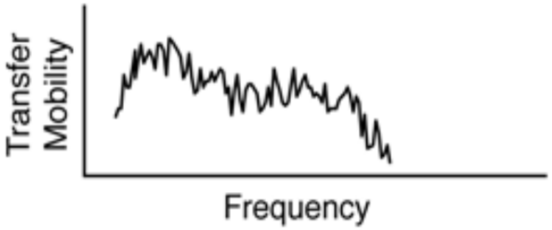
1/3 Octave Band Transfer Mobility
(one spectrum for each distance at each site)

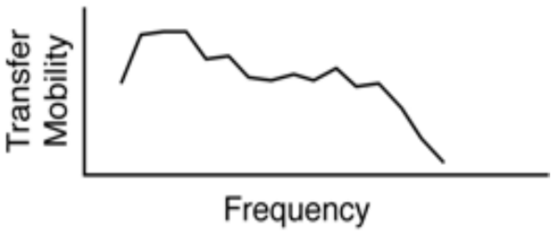
↓

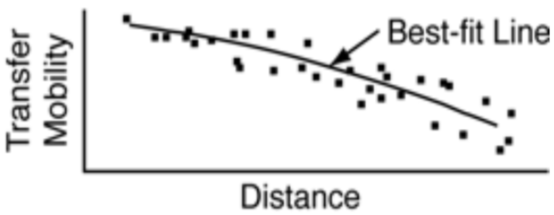
Best-fit Curve of Transfer Mobility as a Function of Distance
(one for each 1/3 octave band)

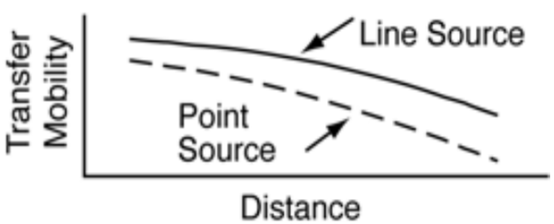
↓

Line-Source Transfer Mobility
(estimated using numerical integration of point source transfer mobility)









MEASUREMENT OF TM

- Relation between force *F* and velocity *V*

$$V\left(\mathbf{r}_2, f\right) = Y\left(\mathbf{r}_2 \mid \mathbf{r}_1, f\right) F(\mathbf{r}_1, f)$$

$$Y\left(\mathbf{r}_2 \mid \mathbf{r}_1, f\right) = \frac{\left\langle G_{FV}\left(f\right)\right\rangle}{\left\langle G_{FF}\left(f\right)\right\rangle} = \text{TRANSFER MOBILITY}$$

$$\left\langle G_{FV}\left(f\right)\right\rangle = \text{CROSS SPECTRUM BETWEEN}$$

$$\left\langle G_{FF}\left(f\right)\right\rangle = \text{AUTO SPECTRUM FORCE}$$

$$\left\langle \quad \right\rangle = \text{EXPECTATION VALUE}$$

COHERENCE

- The coherence is the portion of vibration response energy that is related to the force

$$\gamma^2 = \frac{\langle G_{FV} \rangle \langle G_{FV} \rangle^*}{\langle G_{FF} \rangle \langle G_{VV} \rangle} = \frac{\left| G_{FV} \right|^2}{\langle G_{FF} \rangle \langle G_{VV} \rangle}$$

- The unrelated energy is noise energy that is a source of random uncertainty
- The coherence provides a measure of the uncertainty

STANDARD DEVIATION

$$s.d.\left[Y\right] = \sqrt{\operatorname{var}\left(\operatorname{Re}Y\right) + \operatorname{var}(\operatorname{Im}Y)}$$

$$\operatorname{var}\left(\operatorname{Re}Y\right) = \frac{1}{N_d}\sum_{N_d}\left(\operatorname{Re}Y_i - \operatorname{Re}\bar{Y}\right)^2$$

$$\operatorname{var}\left(\operatorname{Im}Y\right) = \frac{1}{N_d}\sum_{N_d}\left(\operatorname{Im}Y_i - \operatorname{Im}\bar{Y}\right)^2$$

N_d = Number of conversions (or samples)

STANDARD ERROR OF THE MEAN

$$s.e.\left[Y\right] = \frac{s.d.\left[Y\right]}{\sqrt{N_d}} = \frac{\sqrt{\operatorname{var}\left(\operatorname{Re} Y\right) + \operatorname{var}(\operatorname{Im} Y)}}{\sqrt{N_d}}$$

$$N_d = \text{Number of conversions (or samples)}$$

NORMALIZED ERROR OF MEAN COMPLEX MOBILITY

$$\varepsilon\left[Y\right] = \frac{\text{s.e.}\left[Y\right]}{\left|Y\right|}$$

$$\varepsilon\left[Y\right] = \sqrt{\frac{1-\gamma^2}{(N_d - 1)\gamma^2}}$$

$\gamma^2 =$ COHERENCE

$N_D =$ NUMBER OF SAMPLES (CONVERSIONS)

NORMALIZED ERROR OF TRANSFER MOBILITY MAGNITUDE

- Bendat, JSV v59(3) 1978

$$\varepsilon_r\left[\left|Y\right|\right]=\sqrt{\frac{1-\gamma^2}{2N_d\gamma^2}}$$

$$\gamma^2 = \text{COHERENCE}$$

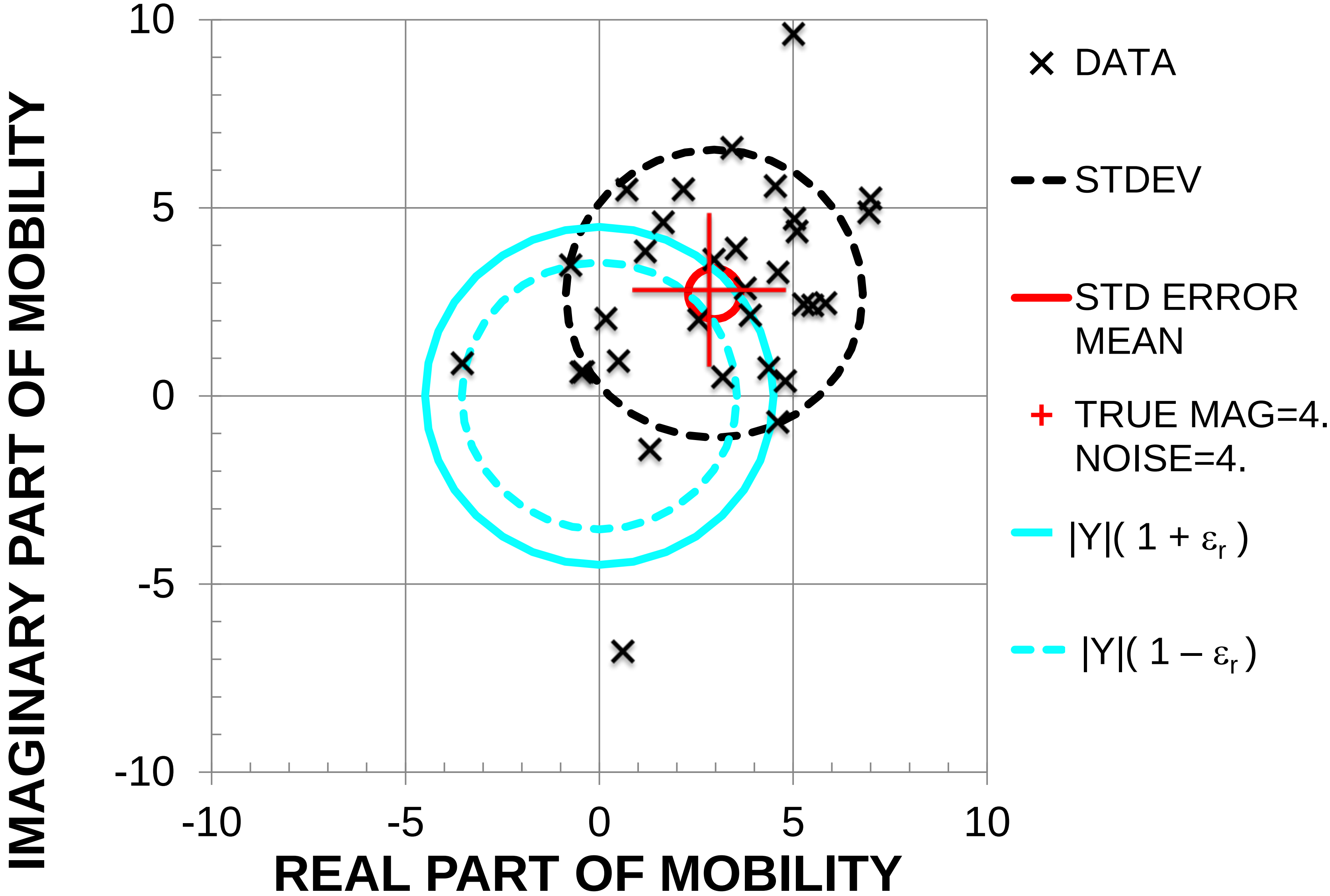
$$N_D = \text{NUMBER OF SAMPLES (CONVERSIONS)}$$

- Note: $\varepsilon\left[Y\right] > \varepsilon_r\left[\left|Y\right|\right]$

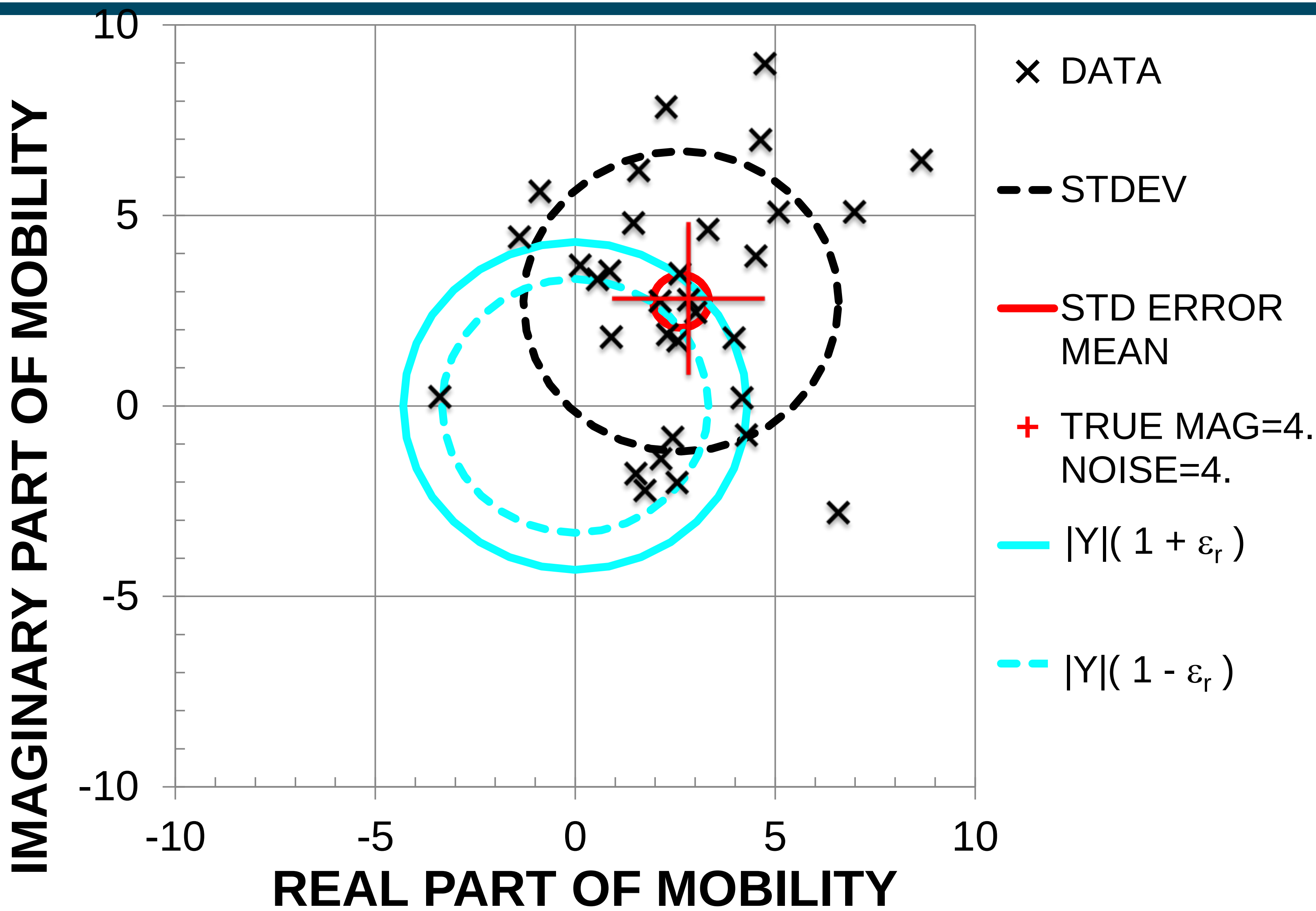
SIMULATION

- Magnitude $|Y| = 4$
- RMS Noise = 4
- Unit force magnitude
  - |GAA| = 1
- 32 Conversions (eg Hammer Hits)

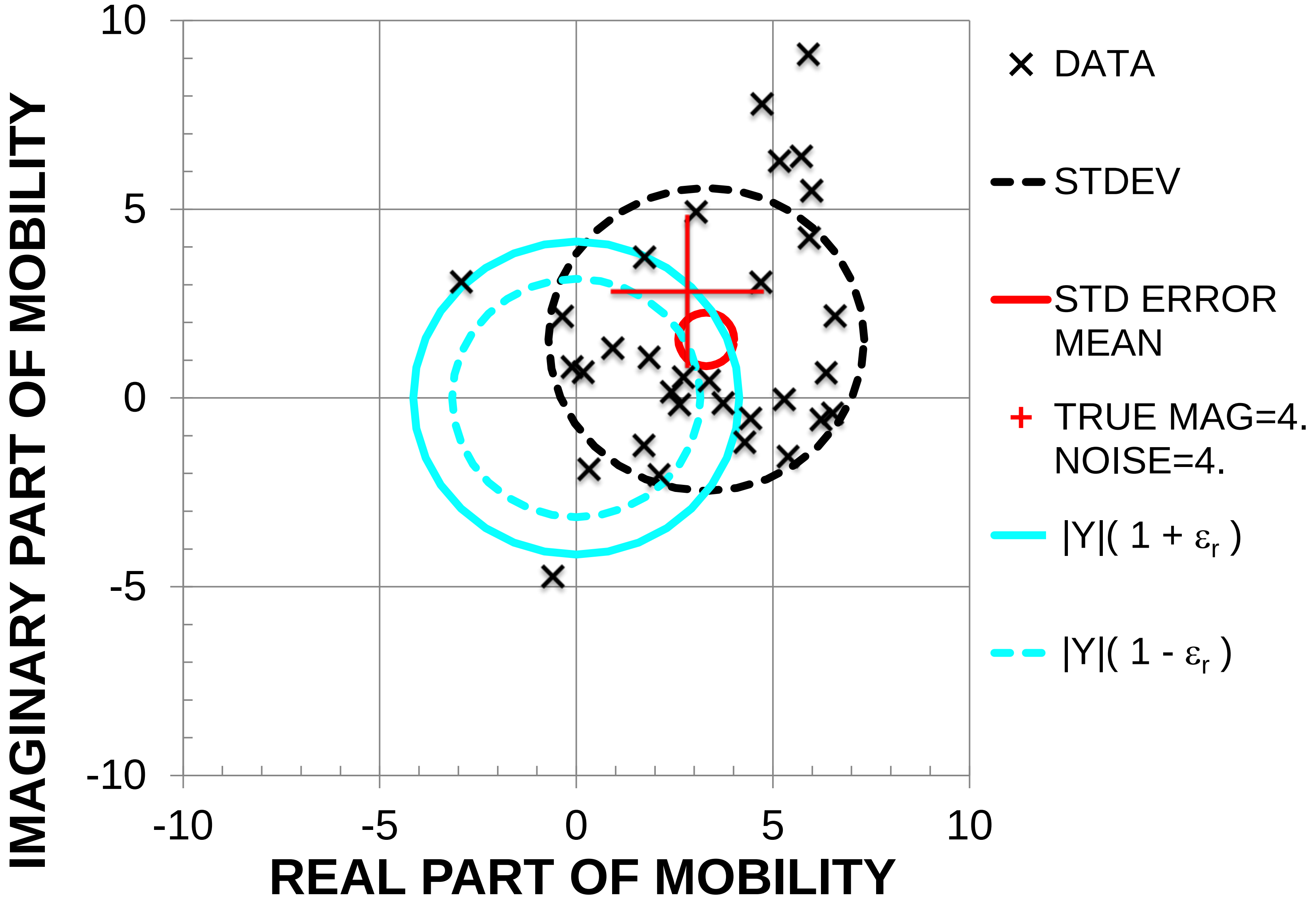
STDEV REAL=2.49 STDEV IMAG=2.91
COHERENCE = .532 TM EST ERR= 12%



STDEV REAL=2.43 STDEV IMAG=3.11
COHERENCE = .491 TM EST ERR= 13%



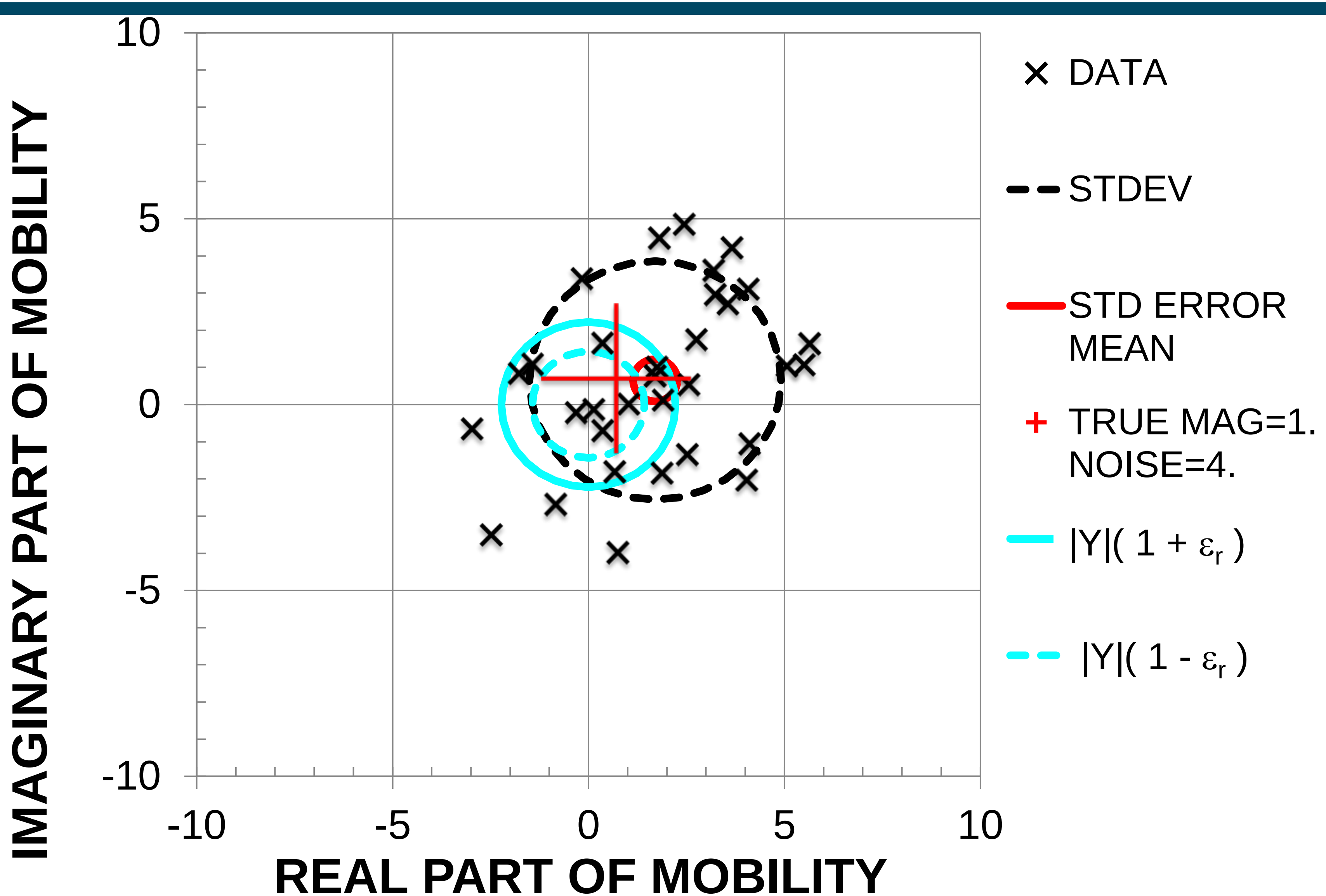
STDEV REAL=2.52 STDEV IMAG=3.12
COHERENCE = .461 TM EST ERR= 14%



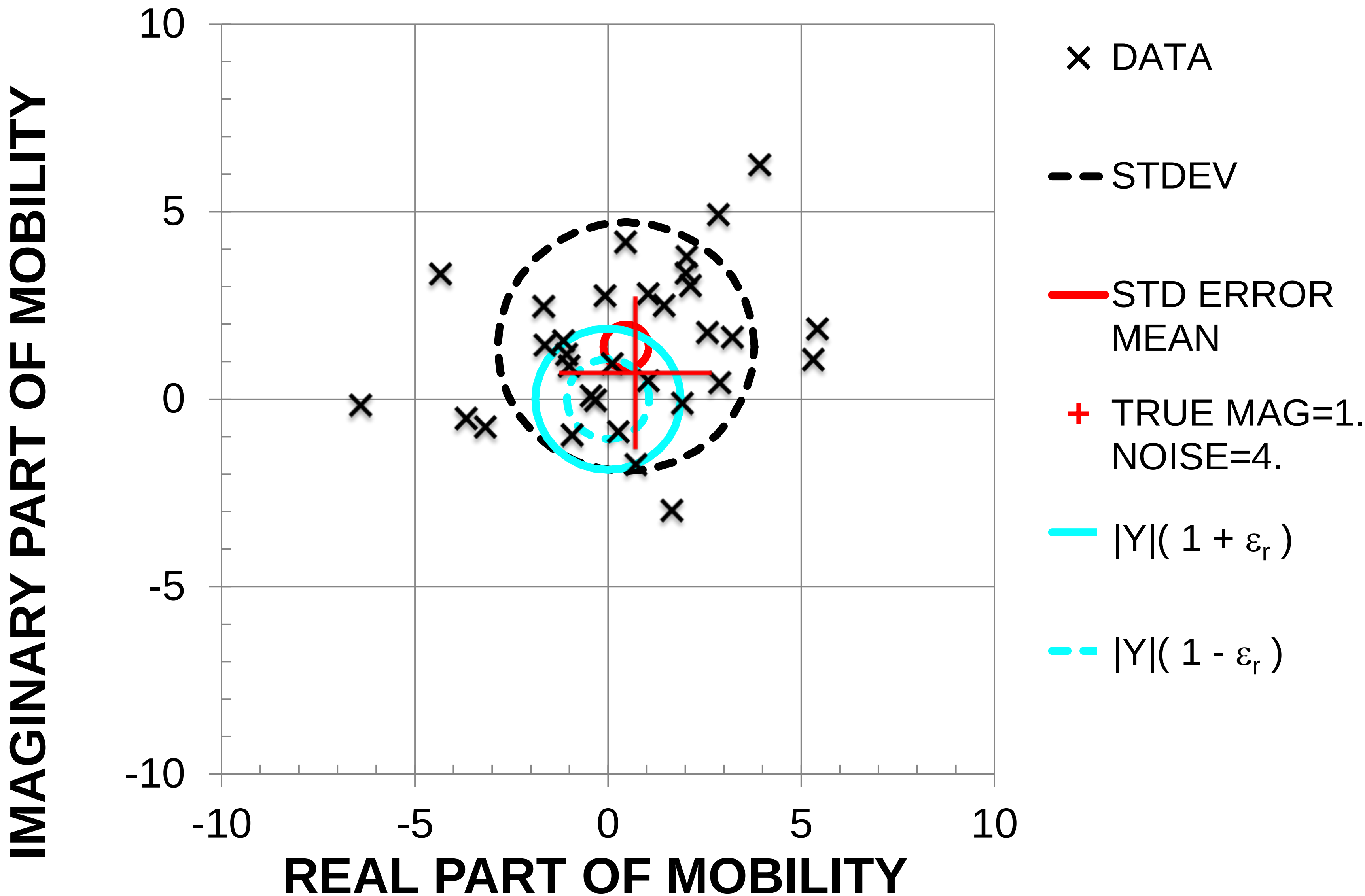
SIMULATION

- Reduce Magnitude $|Y|$ to = 1
- Keep Noise = 4
- Unit force magnitude
  - |GAA| = 1 for all frequencies
- 32 Conversions

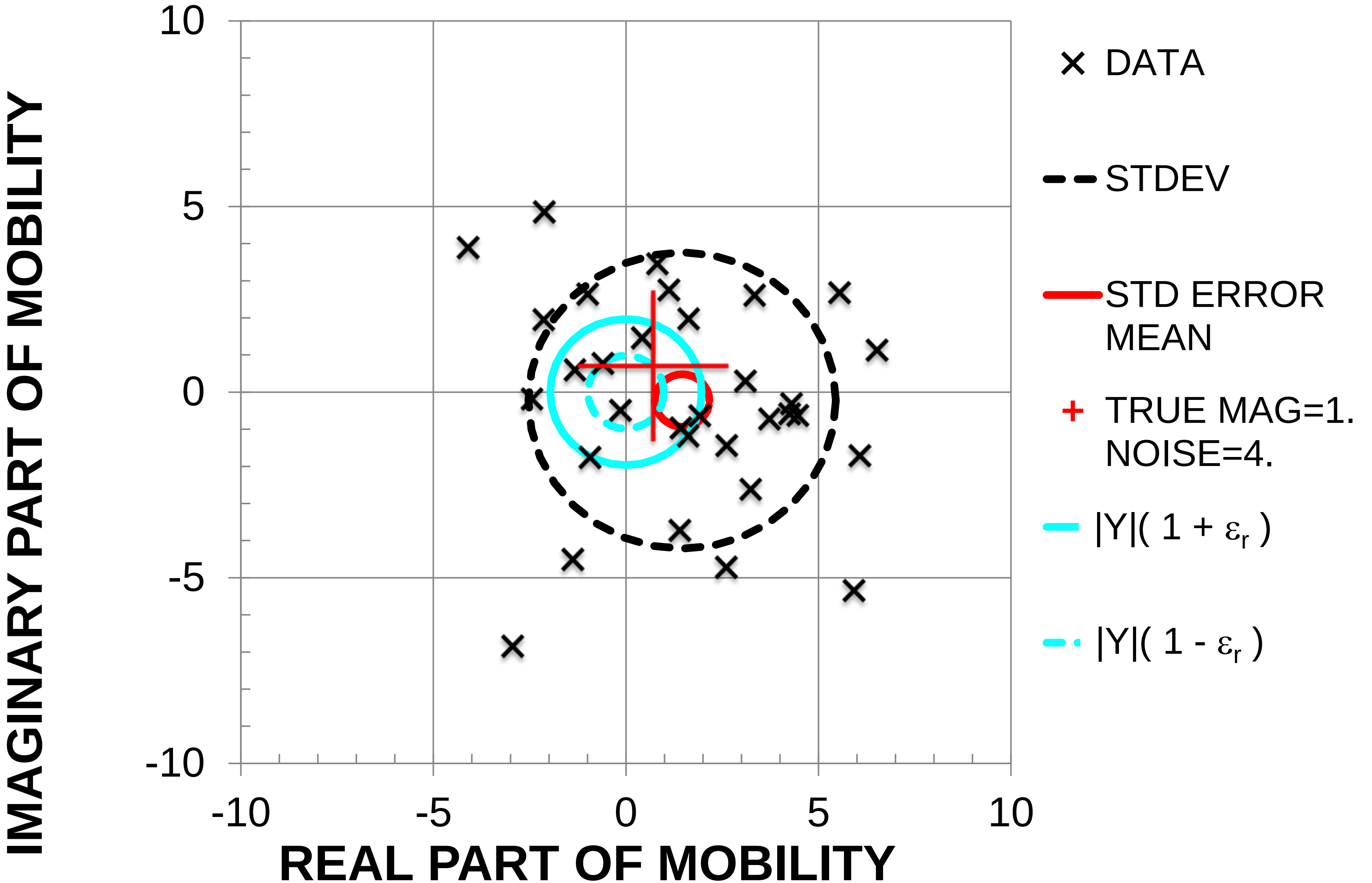
STDEV REAL=2.25 STDEV IMAG=2.29
COHERENCE = .25 TM EST ERR= 22%



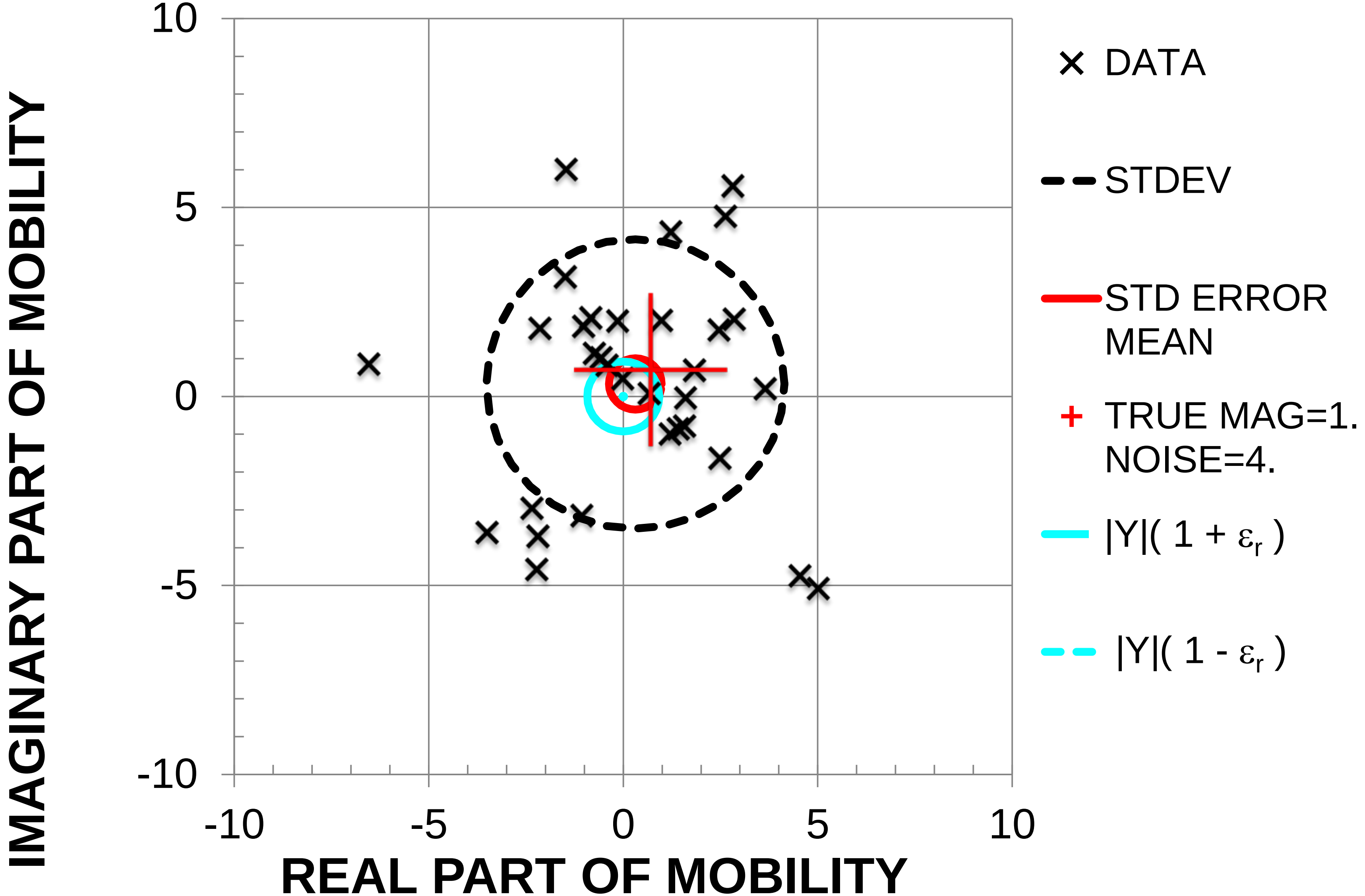
STDEV REAL=2.65 STDEV IMAG=2.01
COHERENCE = .169 TM EST ERR= 28%



STDEV REAL=2.86 STDEV IMAG=2.79
COHERENCE = .124 TM EST ERR= 33%



STDEV REAL=2.47 STDEV IMAG=2.93
COHERENCE = .014 TM EST ERR= 103%



CONVERSION TO 1/3 OCTAVE MOBILITY LEVEL

- Average the square of the magnitude of Y over the 1/3 octave

$$TM_{1/3}(f) = \sqrt{\frac{\int \left|H(f)\right|^2 \left|Y(f)\right|^2 df}{\int \left|H(f)\right|^2 df}} = \text{ 1/3 octave mobility}$$

$H(f) =$ FILTER RESPONSE

$f =$ FREQUENCY

- Energy averaging!

ESTIMATED NORMALIZED ERROR OF $TM_{1/3}$

$$\Sigma = \frac{\langle \varepsilon_r \rangle}{\sqrt{M}} \quad = \text{ Estimated Error of } TM_{1/3}$$

$$\langle \varepsilon_r \rangle = \text{ Mean square estimated error of } |Y| \text{ for each bin}$$

$$M = \text{Number of frequency bins in the 1/3 octave band}$$

$$\Sigma = \frac{1}{\sqrt{M}} \sqrt{\frac{1-\gamma^2}{2N_d\gamma^2}} \quad \text{if all standard errors are the same}$$

REGRESSION vs DISTANCE

- Develop regression curve for integration of the Transfer Mobility over the train
- Most common
  - Level (dB) vs log distance
  - Level (dB) vs polynomial of log distance
  - Level (dB) vs log distance and distance

$$PSR(dB) = 20\log_{10}\left\lfloor TM_{1/3} \right\rfloor$$

REGRESSION METHODS

- Linear Least Squares
- Chi-square minimization

$$\chi^2 = \sum_L \left(\frac{1}{\sigma_i^2} \left\{ PSR_i - \sum_{j=1}^{L} a_j X_j(x_i) \right\}^2 \right)$$

$$\sigma_i(dB) = 20\text{Log}_{10}\left[1 + \Sigma_i\right] = \text{decibel uncertainty at each } x_i$$

x_i distances

$PSR_i =$ transfer mobility level (dB) at x_i

$\sum_{j=1}^{L} a_j X_j(x) =$ regression function of x

$a_j =$ regression coefficients

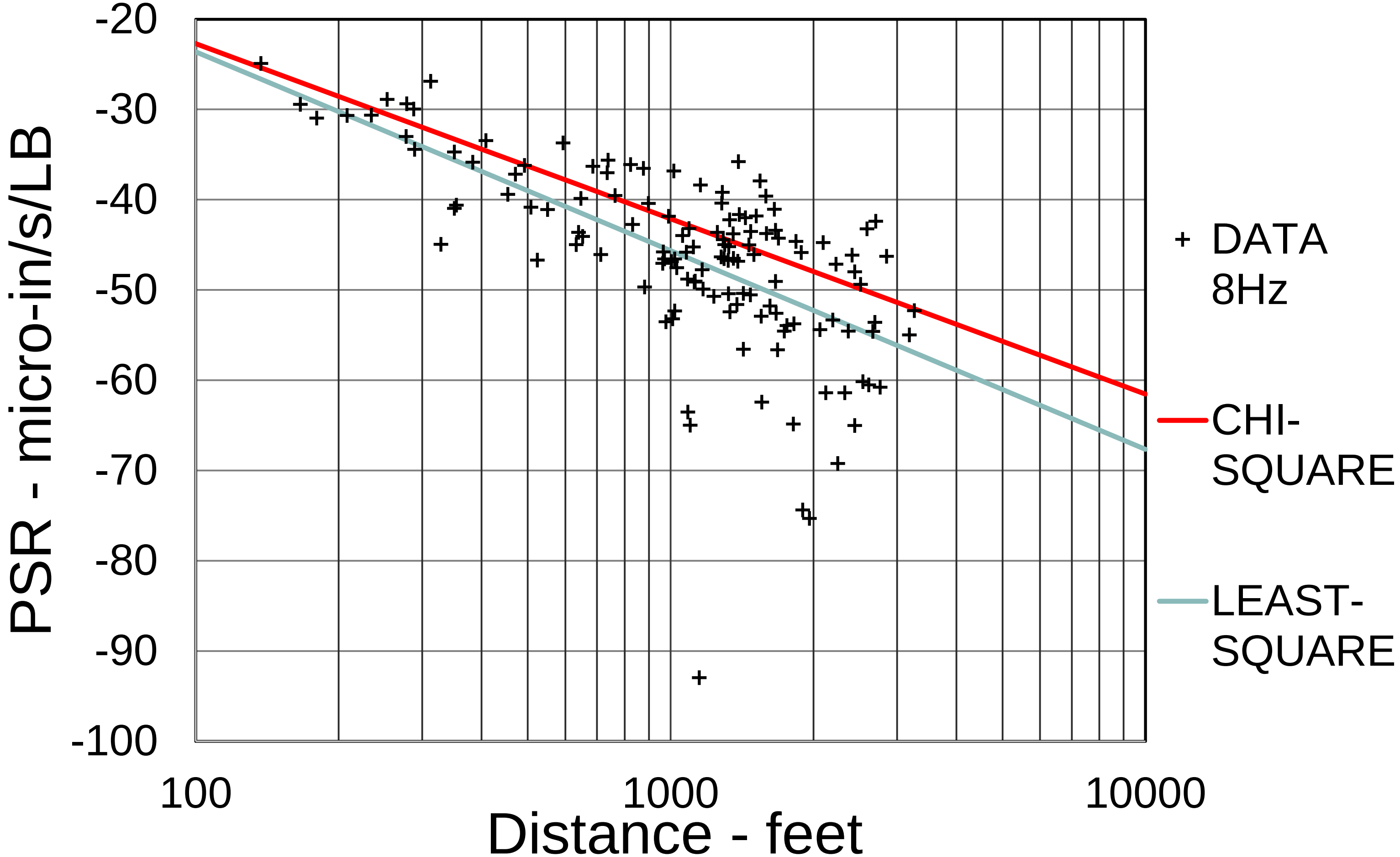
LEVEL UNCERTAINTY IN DB

- Practical relation for 1/3 Octave decibel uncertainty for chi-square regression

$$\sigma\left(dB\right)=20\mathrm{Log}_{10}\left[1+\Sigma_{1/3}\right]$$

EXAMPLE COMPARISON

- Linear regression vs chi-square regression



CONCLUSION

- Transfer mobility analysis with FFT provides sufficient data for estimation of error
  - Coherence function
  - Number of samples
  - Integration over 1/3 octave
  - Regression / Summation
- Easily incorporated into analysis software
- Risk assessment

QUESTIONS